

Multiple Regression: Moderation

PSY 5102: Advanced Statistics for
Psychological and Behavioral Research 2

Goals

- ◉ Introduce moderated multiple regression
 - Continuous predictor \times continuous predictor
 - Continuous predictor \times categorical predictor
- ◉ Understand how to conduct a moderated multiple regression using SPSS
- ◉ Understand how to interpret moderated multiple regression
- ◉ Learn to generate predicted values for interactions using Excel
- ◉ Learn to run simple slopes tests in SPSS
- ◉ Learn how to test higher-order interactions

Going Beyond Simple Associations

- ◉ When research in an area is just beginning, attention is usually devoted to determining whether there is a simple relationship between X and Y (e.g., playing violent video games and engaging in aggressive behavior)
- ◉ The next step is often to determine whether the association is causal or merely an artifact of some kind (e.g., poor research design, poor measurement)
- ◉ Researchers are often not completely satisfied with just demonstrating simple associations

The Importance of Moderation and Mediation

- ◉ As areas of research develop and mature, researchers often turn their attention toward understanding **when** certain associations emerge (under what conditions?) as well as **how** those associations emerge (what are the mechanisms involved?)
- ◉ Answering such questions of when and how results in a deeper understanding of the phenomenon or process being investigated and may provide insights into how that understanding can be applied
- ◉ These kinds of questions are addressed by moderation (when?) and mediation (how?)

The Goal of Moderation

- ◉ A researcher using moderation is typically interested in determining whether the size or sign of the association between X and Y depends on (i.e., interacts with) one or more moderator variables
 - Example: Does the effect of violent video game play on later aggressive behavior depend on trait aggressiveness?
 - Trait aggressiveness may moderate the association between playing violent video games and later aggressive behavior (e.g., playing violent video games may only lead to later aggressive behavior for individuals with high levels of trait aggressiveness)

The Goal of Mediation

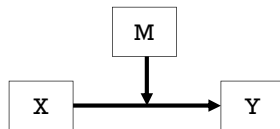
- ◉ The goal of a mediational analysis is to establish the extent to which some putative causal variable (X) influences some outcome variable (Y) through one or more mediator variables
 - Example: Is the association between playing violent video games and later aggressive behavior due to the fact that playing violent video games desensitizes individuals to the pain that others feel?
 - That is, does desensitization explain the association between playing violent video games and later aggressive behavior?
Video games → Desensitization → Aggression

Conditional Process Analysis

- Researchers have more recently come to appreciate that an analysis focused on answering only **when** or **how** questions is likely going to be at least somewhat incomplete
- A more nuanced understanding of a phenomenon often involves uncovering and describing the contingencies of mechanisms (i.e., the when of the how)
- This sort of analysis is referred to by names such as **conditional process analysis**, **moderated mediation**, and **mediated moderation**
 - Example: Is the extent to which the influence of playing violent games on later aggressive behavior through the process of desensitization dependent on trait aggressiveness?
 - That is, do individuals with high levels of trait aggressiveness become desensitized to the feelings of others when they play violent video games which may, in turn, promote later aggressive behavior?
- We will start by focusing on moderation which will be followed by mediation and conditional process analysis

What is a moderator effect?

- Effect of a predictor variable (X) on an outcome variable (Y) depends on a third variable (M)
- The term **moderation** is synonymous with **interaction effect**



Why is Moderation Important?

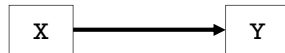
- Most effects that scientists study are contingent on one thing or another
 - An effect may be large for women but small for men
 - An effect may be positive for individuals who are extraverted but negative for individuals who are introverted
- A moderator variable changes the strength of an association between two other variables
 - Moderators indicate **when** or **under what conditions** a particular association can be expected

Examples

- Work stress increases drinking problems for people with a highly avoidant coping style (e.g., denial) but work stress is not associated with drinking problems for individuals with low scores for avoidant coping style (Cooper, Russell, & Frone, 1990)
- Disagreements with friends are associated with drinking at home for college students who report that they drink to cope (e.g., to forget about problems) but these sorts of disagreements are not related to drinking at home for students who do not drink to cope (Mohr et al., 2005)
- Effect of presence of others on performance depends on the dominance of response tendencies (i.e., social facilitation; Zajonc, 1965)
- Effects of stress on health depends on social support (Cohen & Wills, 1985)
- Effect of provocation on aggression depends on trait aggressiveness (Marshall & Brown, 2006)

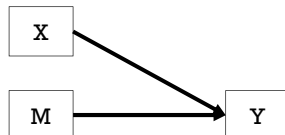
Simple Regression Analysis

$$Y = A + BX + e$$



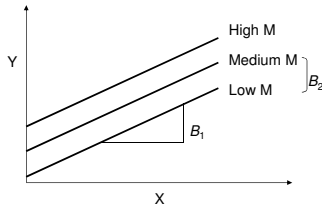
Multiple Regression with Additive Predictor Effects

$$Y = A + B_1X + B_2M + e$$



Multiple Regression with Additive Predictor Effects

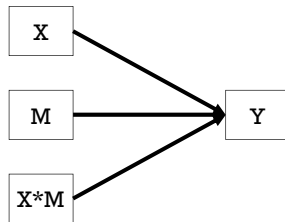
$$Y = A + B_1X + B_2M + e$$



- The **intercept** of the regression of Y on X depends on the specific value of M
- Slope of the regression of Y on X (b_1) stays constant

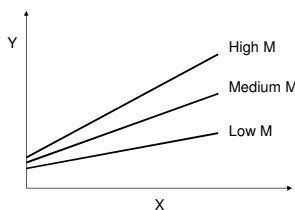
Multiple Regression Including the Interaction of the Predictors

$$Y = A + B_1X + B_2M + B_3X*M + e$$



Multiple Regression Including the Interaction of the Predictors

$$Y = A + B_1X + B_2M + B_3X*M + e$$



- The **slope and intercept** of the regression of Y on X depends on the specific value of M
- There is a different line for every individual value of M (simple regression line)

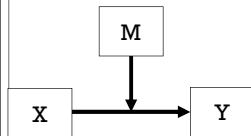
Regression Model with Interaction Term

$$Y = A + B_1X + B_2M + B_3X*M + e$$

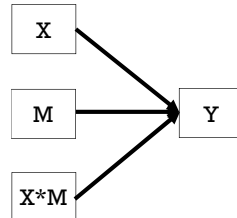
- ◉ The interaction is carried by the $X*M$ term which is the product of X and M
- ◉ The B_3 coefficient reflects the interaction between X and M only if the lower order terms B_1X and B_2M are included in the equation!
 - Leaving out these terms confounds the additive and multiplicative effects which produces misleading results
 - ALWAYS INCLUDE THE MAIN EFFECT TERMS THAT CONSTITUTE YOUR INTERACTION
- ◉ Each individual has a score on X and M
 - To form the $X*M$ term, simply multiply together the individual's scores on X and M

Two Ways of Depicting Moderation

Conceptual Diagram



Statistical Diagram



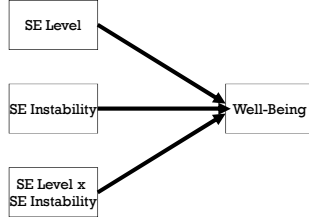
Regression Model with Interaction Term

- ◉ There are two equivalent ways to evaluate whether an interaction is present:
 - Test whether the coefficient B_3 differs significantly from zero
 - Test whether the increment in the squared multiple correlation (ΔR^2) given by the interaction is significantly greater than zero
- ◉ Interactions work with continuous or categorical predictor variables
 - For categorical variables, we have to agree on a coding scheme (dummy vs. effects coding)

Example: Two Continuous Predictors

X: Self-Esteem Level
M: Self-Esteem Instability
Y: Psychological Well-Being

- Does the association between self-esteem level and psychological well-being depend on self-esteem instability?



Moderated Multiple Regression Using SPSS

	rse_tot	seinstab	nwbs_tot	gender
1	2.00	.20	2.89	men
2	2.10	1.42	3.39	women
3	3.40	.46	4.20	men
4	2.70	.50	3.00	women
5	2.60	1.11	3.83	women
6	2.90	.42	3.78	women
7	2.30	1.67	4.42	men
8	4.00	2.25	3.50	women
9	3.10	1.14	3.78	men
10	3.50	1.46	3.72	men
11	5.00	.00	4.39	men
12	3.00	.23	3.81	women
13	2.40	.31	3.33	women
14	3.20	.45	3.78	women
15	3.80	.61	4.89	women
16	3.90	.15	5.00	women
17	4.70	.10	4.78	women
18	3.20	.19	3.28	women
19	3.00	.27	4.56	men

Descriptives

	N	Minimum	Maximum	Mean	Std. Deviation
rse_tot	183	2.00	5.00	4.0095	.76137
seinstab	183	.00	2.48	.5464	.48793
nwbs_tot	183	2.72	6.00	4.6063	.71987
Valid N (listwise)	183				

	rse_tot	seinstab	nwbs_tot
rse_tot	Pearson Correlation	1	-.223 ^{**}
	Sig. (2-tailed)		.002
	N	183	183
seinstab	Pearson Correlation	-.223 ^{**}	1
	Sig. (2-tailed)	.002	.000
	N	183	183
nwbs_tot	Pearson Correlation	.592 ^{**}	-.344 ^{**}
	Sig. (2-tailed)	.000	.000
	N	183	183

**. Correlation is significant at the 0.01 level (2-tailed).

The Problems with Median Splits

- ◉ Why not simply split both X and M into two groups and conduct a Two-Way ANOVA to test for moderation?
 - Disadvantage #1: Median splits are highly sample dependent
 - Disadvantage #2: This will reduce our power to detect interaction effects because we are ignoring useful information about the predictors
 - Disadvantage #3: Median splits can bias the results of moderation

Estimating the Unstandardized Solution

- ◉ **Unstandardized** means that the original metrics of the variables are preserved
- ◉ This is accomplished by **centering** both X and M around their respective sample means
 - Centering refers to subtracting the mean of the variable from each score
 - Centering provides a meaningful zero-point for X and M (gives you effects **at the mean** of X and M, respectively)
 - Centering predictors does not affect the interaction term, but all of the other coefficients in the model
- ◉ Compute crossproduct of cX (centered X) and cM (centered M)
- ◉ $Y = A + B_1cX + B_2cM + B_3cX*cM + e$

SPSS Syntax

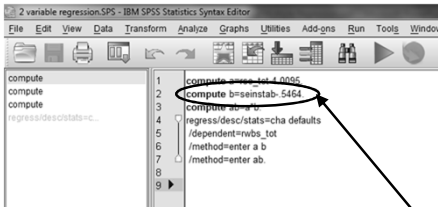
```

1 compute a=rse_tot-4.0095
2 compute c=a-4.0095
3 compute ab=a*b
4 regress /desc/stats=cha defaults
5 /dependent=rwb_tot
6 /method=enter a b
7 /method=enter ab.
8
9

```

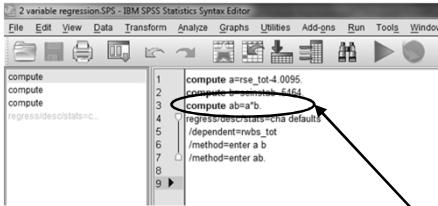
This statement creates variable "a" which will be the centered score for self-esteem level

SPSS Syntax



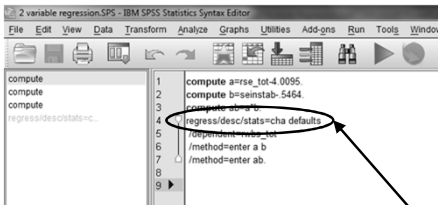
This statement creates variable "b" which will be the centered score for self-esteem instability

SPSS Syntax



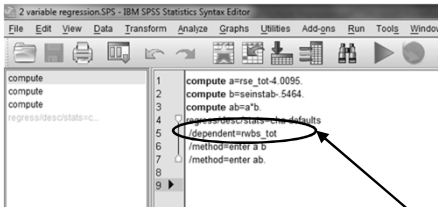
This statement creates the "ab" product term which simply multiplies the centered scores for self-esteem level and self-esteem instability

SPSS Syntax



This statement is the regression command

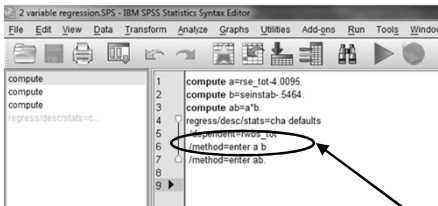
SPSS Syntax



1 compute a=rse_tot*4.0095
2 compute b=seinstab-.5464
3 compute ab=a*b
4 regress/desc/stats=cha defaults
5 /dependent=nwbs_tot
6 /method=enter a b
7 /method=enter ab.
8
9 ▶

This statement identifies psychological well-being as the outcome variable

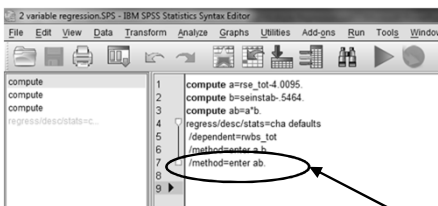
SPSS Syntax



1 compute a=rse_tot*4.0095
2 compute b=seinstab-.5464
3 compute ab=a*b
4 regress/desc/stats=cha defaults
5 /dependent=nwbs_tot
6 /method=enter a b
7 /method=enter ab.
8
9 ▶

This statement is the command that identifies which variables will be entered on the first step of the hierarchical multiple regression

SPSS Syntax



1 compute a=rse_tot*4.0095
2 compute b=seinstab-.5464
3 compute ab=a*b
4 regress/desc/stats=cha defaults
5 /dependent=nwbs_tot
6 /method=enter a b
7 /method=enter ab.
8
9 ▶

This statement is the command that identifies which variable will be entered on the second step of the hierarchical multiple regression

SPSS Output

Descriptive Statistics

	Mean	Std. Deviation	N
rwbs_tot	4.6063	.71887	183
a	.0000	.76137	183
b	.0000	.48793	183
ab	-.0824	.41883	183

Descriptive statistics will be generated for all of the variables in your model

SPSS Output

Correlations

		rwbs_tot	a	b	ab
Pearson Correlation	rwbs_tot	1.000	.592	-.344	-.147
	a	.592	1.000	-.223	.109
	b	-.344	-.223	1.000	-.038
	ab	-.147	.109	-.038	1.000

A correlation matrix will be generated for all of the variables in your model

SPSS Output

Variables Entered/Removed^a

Model	Variables Entered	Variables Removed	Method
1	b, a ^b	.	Enter
2	ab ^b	.	Enter

a. Dependent Variable: rwbs_tot
b. All requested variables entered.

The output will display which variables were entered on each step

Output: Model Summary

Model Summary									
Model	R	R Square	Adjusted R Square	Std. Error of the Estimate	Change Statistics				
					R Square Change	F Change	df1	df2	Sig. F Change
1	.631 ^a	.398	.391	.56086	.398	59.499	2	180	.000
2	.667 ^b	.444	.435	.54028	.046	14.969	1	179	.000

a. Predictors: (Constant), b, a
b. Predictors: (Constant), b, a, ab

This is the R for Model 1 which reflects the degree of association between our first regression model (the main effects of self-esteem level and self-esteem instability) and the criterion variable

Output: Model Summary

Model Summary									
Model	R	R Square	Adjusted R Square	Std. Error of the Estimate	Change Statistics				
					R Square Change	F Change	df1	df2	Sig. F Change
1	.631 ^a	.398	.391	.56086	.398	59.499	2	180	.000
2	.667 ^b	.444	.435	.54028	.046	14.969	1	179	.000

a. Predictors: (Constant), b, a
b. Predictors: (Constant), b, a, ab

This is R^2 for Model 1 which represents the amount of variance in the criterion variable that is explained by the model (SS_{model}) relative to how much variance there was to explain in the first place (SS_{y}). To express this value as a percentage, you should multiply this value by 100 (which will tell you the percentage of variation in the criterion variable that can be explained by the model).

Output: Model Summary

Model Summary									
Model	R	R Square	Adjusted R Square	Std. Error of the Estimate	Change Statistics				
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2	.667 ^b	.444	.435	.54028	.046	14.969	1	179	.000

a. Predictors: (Constant), b, a
b. Predictors: (Constant), b, a, ab

This is the Adjusted R^2 for the first model which corrects R^2 for the number of predictor variables included in the model. It will always be less than or equal to R^2 . It is a more conservative estimate of model fit because it penalizes researchers for including predictor variables that are not strongly associated with the criterion variable

Output: Model Summary

Model Summary									
Model	R	R Square	Adjusted R Square	Std. Error of the Estimate	Change Statistics				
					R Square Change	F Change	df1	df2	Sig. F Change
1	.631 ^a	.398	.391	.56086	.398	59.499	2	180	.000
2	.667 ^b	.444	.435	.54028	.046	14.969	1	179	.000

a. Predictors: (Constant), b, a

b. Predictors: (Constant), b, a, ab

This is the standard error of the estimate for the first model which is a measure of the accuracy of predictions. The larger the standard error of the estimate, the more error in our regression model

Output: Model Summary

Model Summary									
Model	R	R Square	Adjusted R Square	Std. Error of the Estimate	Change Statistics				
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2	.667 ^b	.444	.435	.54028	.046	14.969	1	179	.000

a. Predictors: (Constant), b, a

b. Predictors: (Constant), b, a, ab

The change statistics are not very useful for the first step because it is comparing this model with an empty model (i.e., no predictors)...which is going to be the same as the R^2

Output: Model Summary

Model Summary									
Model	R	R Square	Adjusted R Square	Std. Error of the Estimate	Change Statistics				
					R Square Change	F Change	df1	df2	Sig. F Change
1	.631 ^a	.398	.391	.56086	.398	59.499	2	180	.000
2	.667 ^b	.444	.435	.54028	.046	14.969	1	179	.000

a. Predictors: (Constant), b, a

b. Predictors: (Constant), b, a, ab

This is the R for Model 2 which reflects the degree of association between this regression model (main effects of self-esteem level and self-esteem instability as well as their interaction) and the criterion variable

Output: Model Summary

Model Summary									
Model	R	R Square	Adjusted R Square	Std. Error of the Estimate	Change Statistics				
					R Square Change	F Change	df1	df2	Sig. F Change
1	.631 ^a	.398	.391	.56086	.398	59.499	2	180	.000
2	.667 ^b	.444	.435	.54028	.046	14.969	1	179	.000

a. Predictors: (Constant), b
b. Predictors: (Constant), b, a, ab

This is R^2 for Model 2 which represents the amount of variance in the criterion variable that is explained by the model (SS_y) relative to how much variance there was to explain in the first place (SS_y). To express this value as a percentage, you should multiply this value by 100 (which will tell you the percentage of variation in the criterion variable that can be explained by the model).

Output: Model Summary

Model Summary									
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2	.667 ^b	.444	.435	.54028	.046	14.969	1	179	.000

a. Predictors: (Constant), b, a
b. Predictors: (Constant), b, a, ab

This is the Adjusted R^2 for Model 2 which corrects R^2 for the number of predictor variables included in the model. It will always be less than or equal to R^2 . It is a more conservative estimate of model fit because it penalizes researchers for including predictor variables that are not strongly associated with the criterion variable

Output: Model Summary

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					R Square Change	F Change	df1	df2	Sig. F Change
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2	.667 ^b	.444	.435	.54028	.046	14.969	1	179	.000

a. Predictors: (Constant), b, a
b. Predictors: (Constant), b, a, ab

This is the standard error of the estimate for Model 2 which is a measure of the accuracy of predictions. The larger the standard error of the estimate, the more error in our regression model

Output: Model Summary

Model Summary									
Model	R	R Square	Adjusted R Square	Std. Error of the Estimate	Change Statistics				
					R Square Change	F Change	df1	df2	Sig. F Change
1	.631 ^a	.399	.391	56.621	.044	59.499	2	180	.000 ^b
2	.667 ^b	.444	.435	54.028	.046	14.969	1	179	.000

a. Predictors: (Constant), b, a
b. Predictors: (Constant), b, a, ab

The change statistics tell us whether the addition of the interaction term in Model 2 significantly improved the model fit. A significant "F Change" value means that there has been a significant improvement in model fit (i.e., more variance in the outcome variable has been explained by Model 2 than Model 1)

Output: ANOVA

ANOVA ^a						
Model	Sum of Squares	df	Mean Square	F	Sig.	
1 Regression	37.432	2	18.716	59.499	.000 ^b	
1 Residual	56.621	180	.315			
1 Total	94.053	182				
2 Regression	41.802	3	13.934	47.734	.000 ^c	
2 Residual	52.251	179	.292			
2 Total	94.053	182				

a. Dependent Variable: rwbs_tot
b. Predictors: (Constant), b, a
c. Predictors: (Constant), b, a, ab

The ANOVA for Model 1 tells us whether the model, overall, results in a significantly good degree of prediction of the outcome variable.

Output: ANOVA

ANOVA ^a						
Model	Sum of Squares	df	Mean Square	F	Sig.	
1 Regression	37.432	2	18.716	59.499	.000 ^b	
1 Residual	56.621	180	.315			
1 Total	94.053	182				
2 Regression	41.802	3	13.934	47.734	.000 ^c	
2 Residual	52.251	179	.292			
2 Total	94.053	182				

a. Dependent Variable: rwbs_tot
b. Predictors: (Constant), b, a
c. Predictors: (Constant), b, a, ab

The ANOVA for Model 2 tells us whether the model, overall, results in a significantly good degree of prediction of the outcome variable. However, the ANOVA does not tell us about the individual contribution of variables in the model

Output: Regression Coefficients

Coefficients^a

Model		Unstandardized Coefficients		Standardized Coefficients	t	Sig.
		B	Std. Error	Beta		
1	(Constant)	4.606	.041		111.102	.000
	a	.512	.056	.542	9.140	.000
	b	-.329	.087	-.224	-3.768	.000
2	(Constant)	4.576	.041		112.376	.000
	a	.534	.054	.565	9.835	.000
	b	-.334	.084	-.227	-3.966	.000
	ab	-.372	.096	-.217	-3.869	.000

a. Dependent Variable: rwbs_tot

This is the Y-intercept for Model 1. This means that when a (centered self-esteem level) and b (centered self-esteem instability) are 0 (i.e., those variables are at their means), then we would expect the score for psychological well-being to be 4.606

Output: Regression Coefficients

Coefficients^a

Model		Unstandardized Coefficients		Standardized Coefficients	t	Sig.
		B	Std. Error	Beta		
1	(Constant)	4.606	.041		111.102	.000
	a	.512	.056	.542	9.140	.000
	b	-.329	.087	-.224	-3.768	.000
2	(Constant)	4.576	.041		112.376	.000
	a	.534	.054	.565	9.835	.000
	b	-.334	.084	-.227	-3.966	.000
	ab	-.372	.096	-.217	-3.869	.000

a. Dependent Variable: rwbs_tot

This t-test compares the value of the Y-intercept with 0. If it is significant, then it means that the value of the Y-intercept (4.606 in this example) is significantly different from 0

Output: Regression Coefficients

Coefficients^a

Model		Unstandardized Coefficients		Standardized Coefficients	t	Sig.
		B	Std. Error	Beta		
1	(Constant)	4.606	.041		111.102	.000
	a	.512	.056	.542	9.140	.000
	b	-.329	.087	-.224	-3.768	.000
2	(Constant)	4.576	.041		112.376	.000
	a	.534	.054	.565	9.835	.000
	b	-.334	.084	-.227	-3.966	.000
	ab	-.372	.096	-.217	-3.869	.000

a. Dependent Variable: rwbs_tot

This is the unstandardized slope or gradient coefficient. It is the change in the outcome associated with a one unit change in the predictor variable. This means that for every 1 point that self-esteem level is increased the model predicts an increase of 0.512 for psychological well-being

Output: Regression Coefficients

Coefficients ^a					
Model		Unstandardized Coefficients		Standardized Coefficients	Sig.
		B	Std. Error	Beta	
1	(Constant)	4.606	.041		.111.102
	a	.512	.056	.542	.9.140
	b	-.329	.087	-.224	-.3.768
2	(Constant)	4.576	.041		.112.376
	a	.534	.054	.565	.9.835
	b	-.334	.084	-.227	-.3.966
	ab	-.372	.096	-.217	-.3.869

a. Dependent Variable: rwbs_tot

This is the standardized regression coefficient (β). It represents the strength of the association between the predictor and the criterion variable

Output: Regression Coefficients

Coefficients ^a					
Model		Unstandardized Coefficients		Standardized Coefficients	Sig.
		B	Std. Error	Beta	
1	(Constant)	4.606	.041		.111.102
	a	.512	.056	.542	.9.140
	b	-.329	.087	-.224	-.3.768
2	(Constant)	4.576	.041		.112.376
	a	.534	.054	.565	.9.835
	b	-.334	.084	-.227	-.3.966
	ab	-.372	.096	-.217	-.3.869

a. Dependent Variable: rwbs_tot

This t-test compares the magnitude of the standardized regression coefficient (β) with 0. If it is significant, then it means that the value of β (0.542 in this example) is significantly different from 0 (i.e., the predictor variable is significantly associated with the criterion variable)

Output: Regression Coefficients

Coefficients ^a					
Model		Unstandardized Coefficients		Standardized Coefficients	Sig.
		B	Std. Error	Beta	
1	(Constant)	4.606	.041		.111.102
	a	.512	.056	.542	.9.140
	b	-.329	.087	-.224	-.3.768
2	(Constant)	4.576	.041		.112.376
	a	.534	.054	.565	.9.835
	b	-.334	.084	-.227	-.3.966
	ab	-.372	.096	-.217	-.3.869

a. Dependent Variable: rwbs_tot

This is the unstandardized slope or gradient coefficient. It is the change in the outcome associated with a one unit change in the predictor variable. This means that for every 1 point that self-esteem instability is increased the model predicts a decrease of 0.329 for psychological well-being

Output: Regression Coefficients

Coefficients ^a					
Model		Unstandardized Coefficients		Standardized Coefficients	Sig.
		B	Std. Error	Beta	
1	(Constant)	4.606	.041		.000
	a	.512	.056	.542	.000
	b	-.329	.087	-.224	.000
2	(Constant)	4.576	.041		.000
	a	.534	.054	.565	.000
	b	-.334	.084	-.227	.000
	ab	-.372	.096	-.217	.000

a. Dependent Variable: rwbs_tot

This is the standardized regression coefficient (β). It represents the strength of the association between the predictor and the criterion variable

Output: Regression Coefficients

Coefficients ^a					
Model		Unstandardized Coefficients		Standardized Coefficients	Sig.
		B	Std. Error	Beta	
1	(Constant)	4.606	.041		.000
	a	.512	.056	.542	.000
	b	-.329	.087	-.224	.000
2	(Constant)	4.576	.041		.000
	a	.534	.054	.565	.000
	b	-.334	.084	-.227	.000
	ab	-.372	.096	-.217	.000

a. Dependent Variable: rwbs_tot

This t-test compares the magnitude of the standardized regression coefficient (β) with 0. If it is significant, then it means that the value of β (-0.224 in this example) is significantly different from 0 (i.e., the predictor variable is significantly associated with the criterion variable)

Output: Regression Coefficients

Coefficients ^a					
Model		Unstandardized Coefficients		Standardized Coefficients	Sig.
		B	Std. Error	Beta	
1	(Constant)	4.606	.041		.000
	a	.512	.056	.542	.000
	b	-.329	.087	-.224	.000
2	(Constant)	4.576	.041		.000
	a	.534	.054	.565	.000
	b	-.334	.084	-.227	.000
	ab	-.372	.096	-.217	.000

a. Dependent Variable: rwbs_tot

This is the Y-intercept for Model 2. This means that when centered self-esteem level, centered self-esteem instability, and their interaction are all 0, then the model predicts that the score for psychological well-being will be 4.576

Output: Regression Coefficients

Coefficients ^a					
Model		Unstandardized Coefficients		Standardized Coefficients	Sig.
		B	Std. Error	Beta	
1	(Constant)	4.606	.041		.111.102
	a	.512	.056	.542	9.140
	b	-.329	.087	-.224	-3.768
2	(Constant)	4.576	.041		112.376
	a	.534	.054	.565	9.835
	b	-.334	.084	-.227	-3.966
	ab	-.372	.096	-.217	-3.869

a. Dependent Variable: rwbs_tot

This t-test compares the value of the Y-intercept for Model 2 with 0. If it is significant, then it means that the value of the Y-intercept (4.576 in this example) is significantly different from 0

Output: Regression Coefficients

Coefficients ^a					
Model		Unstandardized Coefficients		Standardized Coefficients	Sig.
		B	Std. Error	Beta	
1	(Constant)	4.606	.041		.111.102
	a	.512	.056	.542	9.140
	b	-.329	.087	-.224	-3.768
2	(Constant)	4.576	.041		112.376
	a	.534	.054	.565	9.835
	b	-.334	.084	-.227	-3.966
	ab	-.372	.096	-.217	-3.869

a. Dependent Variable: rwbs_tot

This is the regression information for self-esteem level when self-esteem instability and the self-esteem level x self-esteem instability product term are also included in the model

Output: Regression Coefficients

Coefficients ^a					
Model		Unstandardized Coefficients		Standardized Coefficients	Sig.
		B	Std. Error	Beta	
1	(Constant)	4.606	.041		.111.102
	a	.512	.056	.542	9.140
	b	-.329	.087	-.224	-3.768
2	(Constant)	4.576	.041		112.376
	a	.534	.054	.565	9.835
	b	-.334	.084	-.227	-3.966
	ab	-.372	.096	-.217	-3.869

a. Dependent Variable: rwbs_tot

This is the regression information for self-esteem instability when self-esteem level and the self-esteem level x self-esteem instability product term are also included in the model

Output: Regression Coefficients

Coefficients ^a					
Model		Unstandardized Coefficients		Standardized Coefficients	Sig.
		B	Std. Error	Beta	
1	(Constant)	4.606	.041		.000
	a	.512	.056	.542	.000
	b	-.329	.087	-.224	.000
2	(Constant)	4.576	.041		.000
	a	.534	.054	.565	.000
	b	-.334	.084	-.227	.000
	ab	-.372	.096	-.217	.000

a. Dependent Variable: rwbs_tot

This is the unstandardized slope or gradient coefficient for the interaction term

Output: Regression Coefficients

Coefficients ^a					
Model		Unstandardized Coefficients		Standardized Coefficients	Sig.
		B	Std. Error	Beta	
1	(Constant)	4.606	.041		.000
	a	.512	.056	.542	.000
	b	-.329	.087	-.224	.000
2	(Constant)	4.576	.041		.000
	a	.534	.054	.565	.000
	b	-.334	.084	-.227	.000
	ab	-.372	.096	-.217	.000

a. Dependent Variable: rwbs_tot

SPSS calculates the standardized regression coefficient for the product term incorrectly so this value is not quite right...but it is what most people use

Output: Regression Coefficients

Coefficients ^a					
Model		Unstandardized Coefficients		Standardized Coefficients	Sig.
		B	Std. Error	Beta	
1	(Constant)	4.606	.041		.000
	a	.512	.056	.542	.000
	b	-.329	.087	-.224	.000
2	(Constant)	4.576	.041		.000
	a	.534	.054	.565	.000
	b	-.334	.084	-.227	.000
	ab	-.372	.096	-.217	.000

a. Dependent Variable: rwbs_tot

This t-test compares the magnitude of the standardized regression coefficient (β) with 0. If it is significant, then it means that the value of β (-0.217 in this example) is significantly different from 0 (i.e., the interaction term is significantly associated with the criterion variable)

Why the Standardized Regression Coefficients Given by SPSS for the Product Term Are Incorrect

- SPSS takes the z-score of the product (z_{XM}) when calculating the standardized scores
- Except in unusual circumstances, z_{XM} is different from $z_X z_M$ which is the product of the two z-scores in which we are interested

$$z_Y = \beta_1 z_X + \beta_2 z_M + \beta_3 z_{XM} \quad \neq \quad z_Y = \beta_1 z_X + \beta_2 z_M + \beta_3 z_X z_M$$

Incorrect! Correct!

- Friedrich (1982) suggested the following solution: Run a regression using standardized variables

Output: Regression Coefficients for Standardized Variables

Descriptive Statistics

	Mean	Std. Deviation	N
Zscore(wbs_b0)	.067	1.00000000	103
a	.0000	1.000000	103
b	.0000	1.000000	103
ab	-.2217	1.12742	103

Coefficients^a

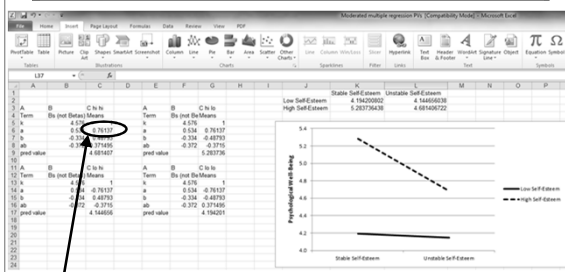
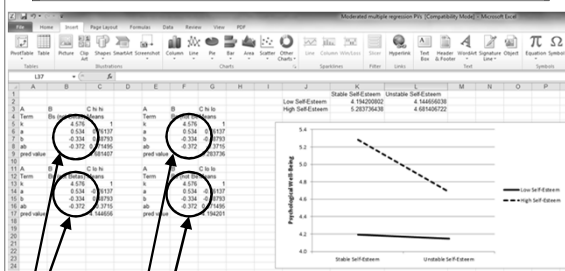
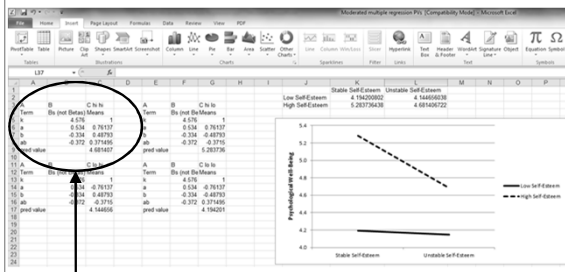
Model		Unstandardized Coefficients	Std. Error	Standardized Coefficients	t	Sig.
1	(Constant)	2.5136-015	.058		.000	1.000
	a	.542	.059	.542	9.140	.000
	b	-.224	.059	-.224	-3.768	.000
2	(Constant)	-.043	.057		-.753	.453
	a	.565	.057	.565	9.835	.000
	b	-.182	.057	-.227	-3.866	.000
	ab	-.192	.050	-.217	-3.869	.000

a. Dependent Variable: Zscore(wbs_b0)

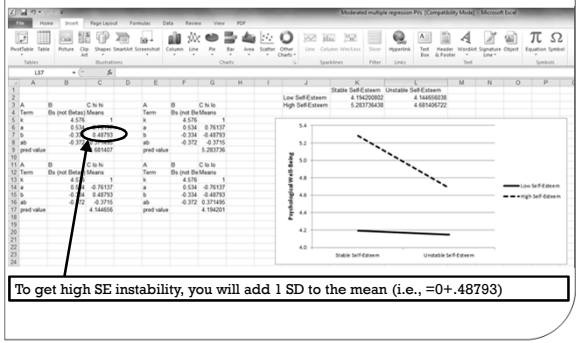
This is the correct standardized regression coefficient for the product term of self-esteem level x self-esteem instability (even though it is in the unstandardized column). NOTE: It is important that the predictors AND the outcome variable are standardized!

Plotting the Interaction

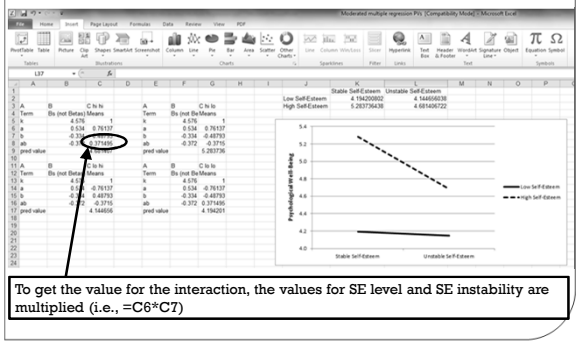
- SPSS does not provide a straightforward module for plotting interactions
 - The modules that are available (e.g., PROCESS) are not flexible enough to easily deal with all of the situations that you may encounter
- There is an infinite number of slopes we could compute for different combinations of X and M
- The most useful is to calculate values for high (+1 SD) and low (-1 SD) X as a function of high (+1 SD) and low (-1 SD) values of M
 - The use of +/- 1SD is simply a convention. There is nothing inherently special about those particular values



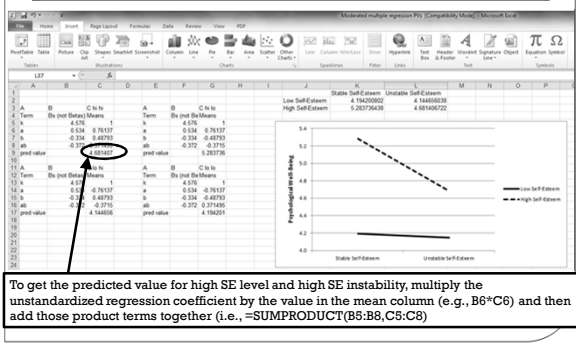
Using Excel to Generate Predicted Values

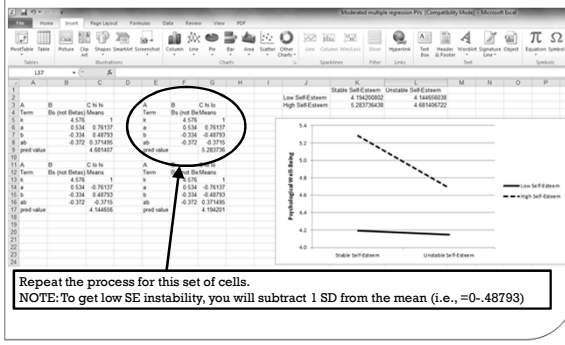


Using Excel to Generate Predicted Values

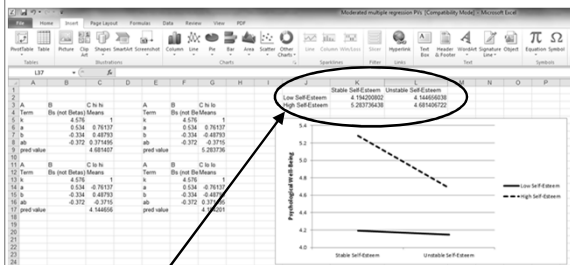


Using Excel to Generate Predicted Values



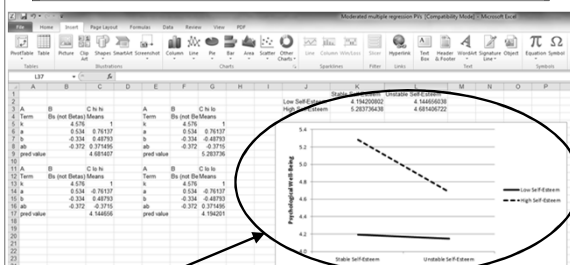


Using Excel to Generate Predicted Values



This spreadsheet is set up to use the predicted values to create the plot. For example, the cell K2 which represents the predicted value for psychological well-being when individuals report low SE level and low SE instability will automatically use the value from G17 (i.e., =G17)

Using Excel to Generate Predicted Values



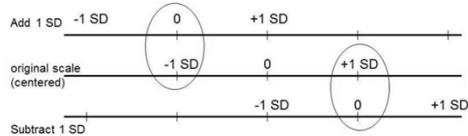
It appears that individuals with high self-esteem (dotted line) generally report higher levels of psychological well-being than those with low self-esteem (solid line) and that this is especially true for those with stable high self-esteem...but we need to test this to make sure

Simple Slopes Testing

- Test of interaction term: **Does the association between X and Y depend on M ?**
- Simple slope testing: **Do the regression coefficients for high (+1 SD) and low (-1 SD) values of X (and M) differ significantly from zero?**
 - This approach is also sometimes referred to as the **pick-a-point approach** or **spotlight analysis**

Simple Slopes Testing

- Simple slope testing for low self-esteem (-1 SD)
 - To test this simple slope, we **+1 SD** to the values of SE level
- Simple slope testing for high self-esteem (+1 SD)
 - To test this simple slope, we **-1 SD** from the values of SE level
- This seems backward but by adding (or subtracting) a standard deviation from each score, we are shifting the zero points
 - Now run separate regression analysis with each transformed score



SPSS Syntax

```

1 compute rse_c=rse_tot-4.0095.
2 compute rse_h=rse_c-.76137.
3 compute rse_l=rse_c+.76137.
4 execute.
5
6 compute a=rse_h.
7 compute b=rse_lab-5454.
8 compute ab=a*b.
9 regress /dep=truis_tot
10 /method=enter a b.
11 /method=enter ab.
12
13 compute a=rse_l.
14 compute b=rse_lab-5454.
15 compute ab=a*b.
16 regress /dep=truis_tot
17 /method=enter a b.
18 /method=enter ab.
19
20

```

Simple Slopes Testing: Results

High SE Level

Coefficients ^a					
Model		Unstandardized Coefficients	Standardized Coefficients	t	Sig.
1	(Constant)	4.995	.059	84.000	.000
	a	.512	.056	9.140	.000
	b	-.329	.087	-3.768	.000
2	(Constant)	4.982	.057	86.773	.000
	a	.534	.054	9.835	.000
	b	-.617	.112	-5.493	.000
	ab	-.372	.096	-3.869	.000

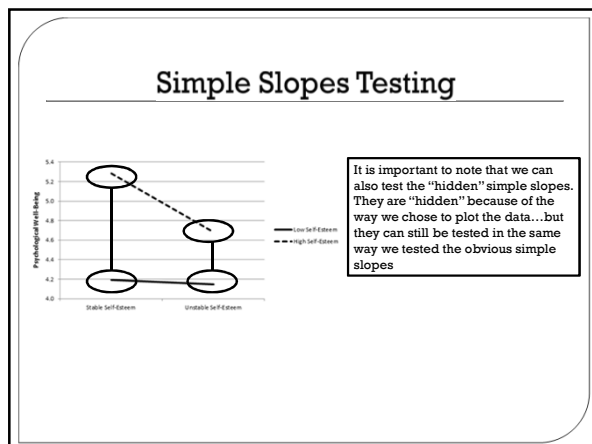
a. Dependent Variable: truis_tot

Low SE Level

Coefficients ^a					
Model		Unstandardized Coefficients	Standardized Coefficients	t	Sig.
1	(Constant)	4.216	.050	70.889	.000
	a	.512	.056	9.140	.000
	b	-.329	.087	-3.768	.000
2	(Constant)	4.169	.059	71.178	.000
	a	.534	.054	9.835	.000
	b	-.651	.111	-5.834	.000
	ab	-.372	.096	-3.869	.000

a. Dependent Variable: truis_tot

We are testing the association between SE instability and psychological well-being when SE level is high and low



[illegible]

Simple Slopes Testing: Results

High SE Instability

Coefficients^a

Model		Unstandardized Coefficients		Standardized Coefficients		t	Sig.
		B	Std. Error	Beta			
1	(Constant)	4.444	.059			74.747	.000
	a	.512	.056	.542		9.140	.000
	b	-.329	.087	-.224		-3.768	.000
2	(Constant)	4.413	.059			76.185	.000
	a	.352	.069	.373		5.177	.000
	b	-.334	.084	-.227		-3.965	.000
	ab	-.372	.096	-.274		-3.869	.000

a. Dependent Variable: nels_1st

We are testing the association between SE level and psychological well-being when SE instability is high and low

Low SE Instability

Coefficients^a

Model		Unstandardized Coefficients		Standardized Coefficients		t	Sig.
		B	Std. Error	Beta			
1	(Constant)	4.787	.059			80.142	.000
	a	.512	.056	.542		9.140	.000
	b	-.329	.087	-.224		-3.768	.000
2	(Constant)	4.739	.059			82.027	.000
	a	.715	.075	.757		9.487	.000
	b	-.334	.084	-.227		-3.965	.000
	ab	-.372	.096	-.305		-3.889	.000

a. Dependent Variable: nels_3rd

Simple Linear Testing: Results

High SE Instability

Coefficients*

Model		Unstandardized Coefficients		Standardized Coefficients		t	Sig.
		B	Std. Error	Beta			
1. (Constant)		4.444	.059			74.747	.000
	a	-.512	.056	-.542		9.140	.000
	b	-.329	.087	-.224		-3.768	.000
2. (Constant)		4.413	.059			74.744	.000
	a	.352	.069	.373		6.137	.000
	ab	-.372	.096	-.274		-3.869	.000

a. Dependent Variable: nels_tot

Low SE Instability

Coefficients*

Model		Unstandardized Coefficients		Standardized Coefficients		t	Sig.
		B	Std. Error	Beta			
1. (Constant)		4.767	.059			80.142	.000
	a	.512	.056	.542		9.140	.000
	b	-.329	.087	-.224		-3.768	.000
2. (Constant)		4.738	.059			80.627	.000
	a	.715	.075	.757		9.467	.000
	b	-.334	.084	-.227		-3.966	.000
3. (Constant)		4.722	.059			80.142	.000
	ab	-.372	.096	-.305		-3.869	.000

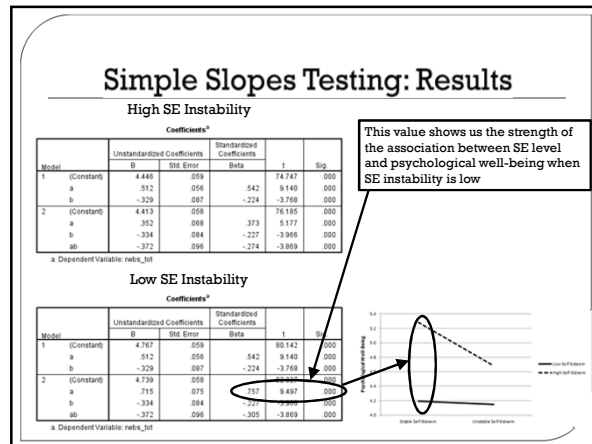
a. Dependent Variable: nels_tot

This value shows us the strength of the association between SE level and psychological well-being when SE instability is high

Psychological Well-being

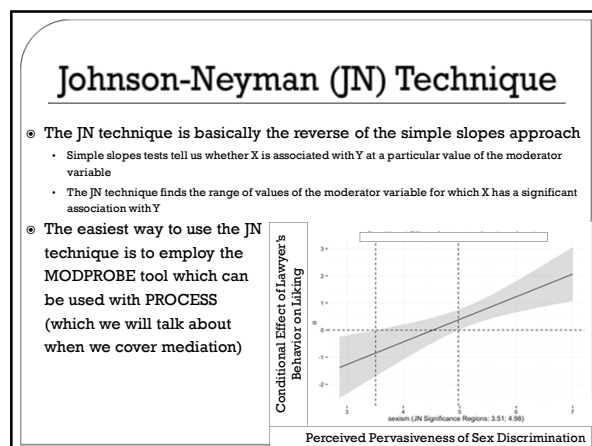
High SE Instability

Low SE Instability



Another Approach to Probing Interactions

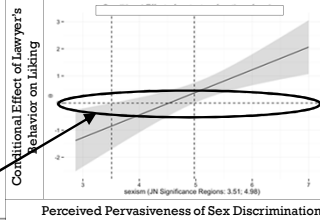
- The simple slopes technique has one major problem. It requires the researcher to select values of the moderator at which to estimate the conditional association between X and Y (e.g., ± 1 SD)
 - Different choices can lead to different claims and the choice of values is often arbitrary (e.g., the selection of ± 1 SD is simply a matter of convention)
- The Johnson-Neyman (JN) technique avoids the selection of specific values of the moderator by determining **regions of significance**
 - This approach is also referred to as a **floodlight analysis** (compared with the spotlight analysis characterized by the simple slopes approach)



Johnson-Neyman (JN) Technique

- The JN technique is basically the reverse of the simple slopes approach
 - Simple slopes tests tell us whether X is associated with Y at a particular value of the moderator variable
 - The JN technique finds the range of values of the moderator variable for which X has a significant association with Y
- The easiest way to use the JN technique is to employ the MODPROBE tool which can be used with PROCESS (which we will talk about when we cover mediation)

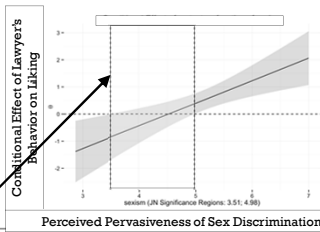
This line represents "no association" between X ("lawyer's behavior" in this example) and Y ("liking" in this example)



Johnson-Neyman (JN) Technique

- The JN technique is basically the reverse of the simple slopes approach
 - Simple slopes tests tell us whether X is associated with Y at a particular value of the moderator variable
 - The JN technique finds the range of values of the moderator variable for which X has a significant association with Y
- The easiest way to use the JN technique is to employ the MODPROBE tool which can be used with PROCESS (which we will talk about when we cover mediation)

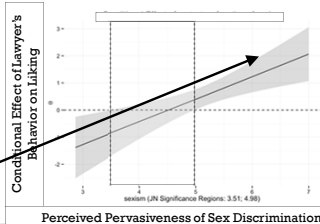
This is the "region of significance" which shows the values of the moderator ("perceived pervasiveness of sex discrimination" in this example) for which X and Y are associated



Johnson-Neyman (JN) Technique

- The JN technique is basically the reverse of the simple slopes approach
 - Simple slopes tests tell us whether X is associated with Y at a particular value of the moderator variable
 - The JN technique finds the range of values of the moderator variable for which X has a significant association with Y
- The easiest way to use the JN technique is to employ the MODPROBE tool which can be used with PROCESS (which we will talk about when we cover mediation)

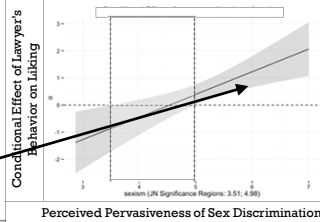
This is the upper limit of the 95% confidence interval



Johnson-Neyman (JN) Technique

- The JN technique is basically the reverse of the simple slopes approach
 - Simple slopes tests tell us whether X is associated with Y at a particular value of the moderator variable
 - The JN technique finds the range of values of the moderator variable for which X has a significant association with Y
- The easiest way to use the JN technique is to employ the MODPROBE tool which can be used with PROCESS (which we will talk about when we cover mediation)

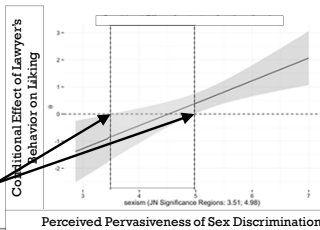
This is the lower limit of the 95% confidence interval



Johnson-Neyman (JN) Technique

- The JN technique is basically the reverse of the simple slopes approach
 - Simple slopes tests tell us whether X is associated with Y at a particular value of the moderator variable
 - The JN technique finds the range of values of the moderator variable for which X has a significant association with Y
- The easiest way to use the JN technique is to employ the MODPROBE tool which can be used with PROCESS (which we will talk about when we cover mediation)

The region of significance is determined by the locations where the upper and lower bounds of the 95% confidence interval intersect 0

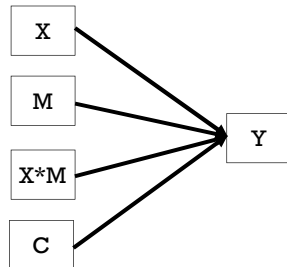


Inclusion of Control Variables

- You may want to control for other variables (i.e., covariates)
- Simply add centered continuous covariates as predictors to the regression equation
- In case of categorical control variables, you may want to use effect coding
 - With effect coding, the categorical variable is analyzed as a set of (non-orthogonal) contrasts which opposes all but one category to another category
 - With effect coding, the intercept is equal to the grand mean and the slope for a contrast expresses the difference between a group and the grand mean
 - Example with 2 categories (single variable)
 - Men (+1), Women (-1)
 - Example with 3 categories (two variables)
 - Democrats (1, 0)
 - Independents (0, 1)
 - Republicans (-1, -1)

Inclusion of a Control Variable

$$Y = A + B_1X + B_2M + B_3X*M + B_4C + e$$



Effect Size Calculation

- ◉ Standardized regression coefficient (β) is already an effect size statistic but it is not perfect
- ◉ f^2 (see Aiken & West, 1991, p. 157)

Calculating f^2

$$f^2 = \frac{r^2_{Y, AI} - r^2_{Y, A}}{1 - r^2_{Y, AI}}$$

$r^2_{Y, AI}$: Squared multiple correlation resulting from combined prediction of Y by the additive set of predictors (A) and their interaction (I) (= full model)

$r^2_{Y, A}$: Squared multiple correlation resulting from prediction by set A only (= model without interaction term)

- ◉ f^2 gives you the proportion of systematic variance accounted for by the interaction relative to the unexplained variance in the outcome variable
- ◉ Conventions by Cohen (1988)
 - $f^2 = .02$: small effect
 - $f^2 = .15$: medium effect
 - $f^2 = .26$: large effect

Example of Effect Size

Model Summary									
Model	R	R Square	Adjusted R Square	Std. Error of the Estimate	Change Statistics				
					R Square Change	F Change	df1	df2	Sig. F Change
1	.631 ^a	.398	.391	.56086	.398	59.499	2	180	.000
2	.667 ^b	.444	.435	.54028	.046	14.969	1	179	.000

a. Predictors: (Constant), b, a
b. Predictors: (Constant), b, a, ab

$$f^2 = \frac{r^2_{Y,AB} - r^2_{Y,A}}{1 - r^2_{Y,A}} = \frac{.444 - .398}{1 - .444} = .083 \rightarrow \text{small effect of interaction}$$

Continuous × Categorical Variable Interaction

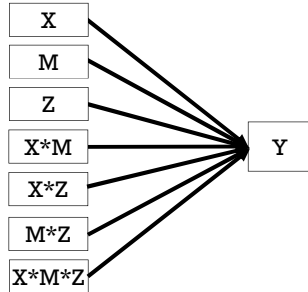
- Very similar process to what we described for continuous × continuous
- The major difference involves the coding scheme for the categorical variable (i.e., effect coding v. dummy coding)
 - We will talk more about coding schemes when we talk about curvilinear regression

Higher-Order Interactions

- Higher-order interactions refers to interactions among more than 2 variables (e.g., a three-way interaction)
- All basic principles (centering, coding, probing, simple slope testing, effect size) generalize to higher-order interactions
- The major difference is that you begin probing the three-way interaction by examining whether the two-way interactions emerge at high and low values of one of your moderators
 - Example: SE level × SE instability × Sex → Well-being
 - I would begin probing the interaction by determining whether the SE level × SE instability interaction emerged for men and women separately...then follow the basic procedures as outlined earlier

Example of a Three-Way Interaction

$$Y = A + B_1X + B_2M + B_3Z + B_4XM + B_5XZ + B_6MZ + B_7XMZ + e$$



Plotting a Three-Way Interaction

- Plot first-level moderator effect (e.g., SE level x SE instability) at different levels of the third variable (e.g., sex)
- It is best to use two graphs with each showing a two-way interaction
- There are 6 different ways to plot the three-way interaction...
- Best presentation should be determined by theory
- In the case of categorical variables (e.g., sex) it often makes sense to plot the separate graphs as a function of group (one graph for men and another for women)
- The logic to compute the values for different combinations of high and low values on predictors is the same as in the two-way case

The Challenge of Statistical Power When Testing Moderator Effects

- If variables were measured without error, the following sample sizes are needed to detect small, medium, and large interaction effects with adequate power (80%)
 - Large effect ($f^2 = .26$): $N = 26$
 - Medium effect ($f^2 = .13$): $N = 55$
 - Small effect ($f^2 = .02$): $N = 392$
- Busemeyer & Jones (1983): reliability of product term of two uncorrelated variables is the product of the reliabilities of the two variables
 - $.80 \times .80 = .64$
- Required sample size is more than doubled when predictor reliabilities drop from 1 to .80
- Problem gets even worse for higher-order interactions