Multilevel Linear Modeling

PSY 5102: Advanced Statistics for Psychological and Behavioral Research 2

Goals

- What is multilevel modeling?
- Examples of multilevel data structures
- Current applications
- Why multilevel modeling?
- What types of studies use multilevel modeling?

Hierarchical Data

- Data structures are often hierarchical
  - This is referred to as a **Nested Data Structure**
- Examples:
  - Children nested within classrooms within schools
  - Data points nested within people
  - Employees nested within organizations
  - Patients nested within hospitals
  - Patients nested within doctors nested within clinics
  - Families within neighborhoods within communities

What Are Nested Data?

- Sub-units are grouped (or "nested") within larger units
  - Many statistical techniques assume that observations are independent…but this assumption is violated with nested data
- Often the data are observations of individuals nested within groups
  - This is important because individuals within groups are often more similar to one another than to individuals in the other groups
  - We can empirically verify this
- Sometimes data are multiple observations nested within an individual

Multilevel Data Structures

Level 4: State

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Level 3: School

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Level 2: Class

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Level 1: Student

Basic Terminology

- A **Level** consists of individual units (e.g., students, classrooms, schools, observations)
- Level-1 variables
  - These are the variables that are nested within groups
  - Example: individual students
- Level-2 variables
  - Typically these are group-level variables
  - Example: classes of students
- There can also be Level-3 or higher variables
  - There must be at least two levels for it to be a multilevel design
- Note that growth models have time periods at Level-1 and individuals at Level-2

Why Multilevel Modeling?

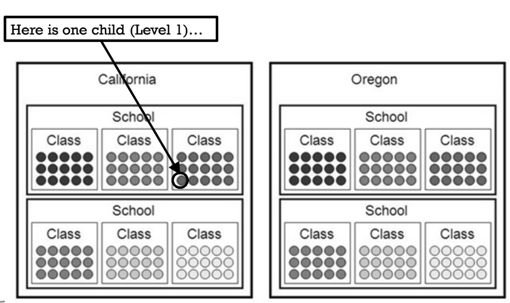
- Nested data are very common in psychology
- Analysis of nested data poses a unit of analysis problem
  - Should we analyze the individual or the group?
  - Unfortunately, we are often unable to choose one over the other
- Traditional linear models offer a simple view of a complex world by generally assuming the same effects across groups
- If effects do differ across groups, we can explain these differences with multilevel modeling

Basic Terminology

- There are multiple terms that are used to refer to Multilevel Modeling
  - Hierarchical Linear Modeling (HLM) is both a statistical technique and a software package
  - Multilevel Random-Coefficient Models
  - Random-Coefficient Regression Models

First Type of Nesting

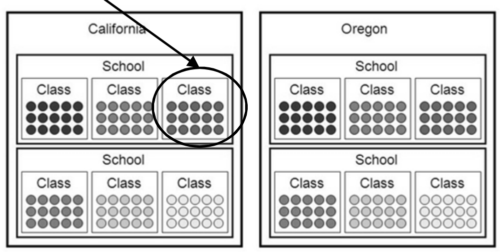
- Individuals Nested Within Groups



First Type of Nesting

- Individuals Nested Within Groups

…who is nested within a particular classroom (Level 2)…



First Type of Nesting

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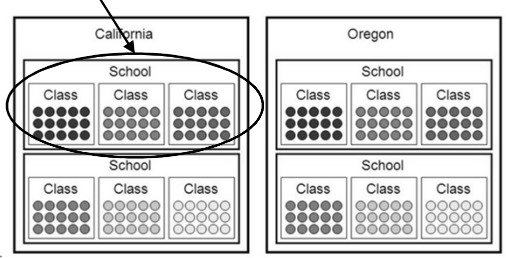
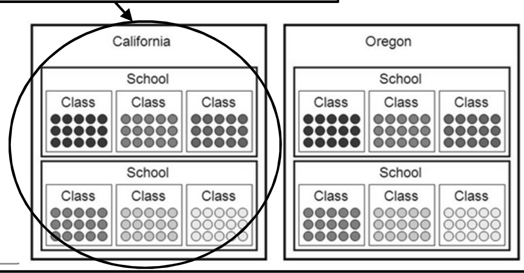
…which is nested within a particular school (Level 3)…



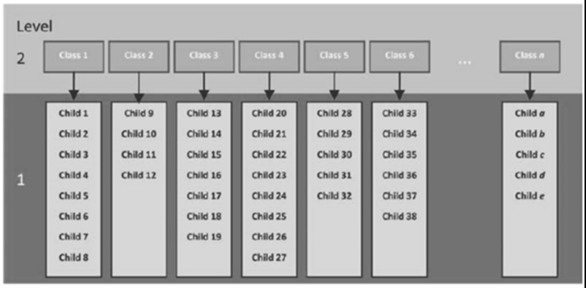
First Type of Nesting

- Individuals Nested Within Groups

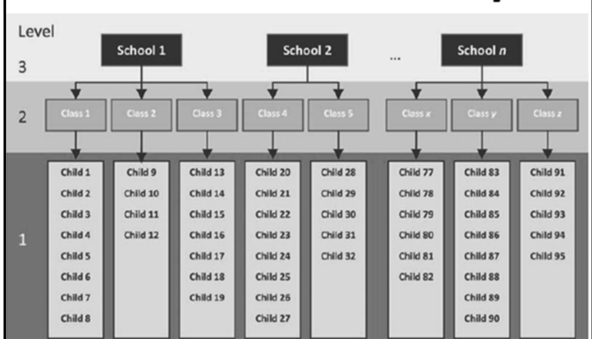
…which is nested within a particular state (Level 4)…



A Two-Level Hierarchy



A Three-Level Hierarchy



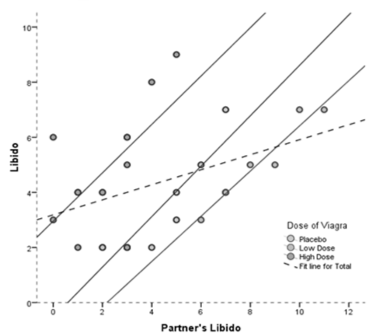
Benefits of Multilevel Models

- It allows you to model the variability in intercepts and regression slopes
- It allows you to account for the associations between cases
  - This allows you to deal with the violation of the independence assumption that accompanies nested data
- Multilevel models can cope with "missing" data
  - It is quite common for nested data to be "uneven" (e.g., some classes may have more children than others, some participants may complete more daily measures than others)

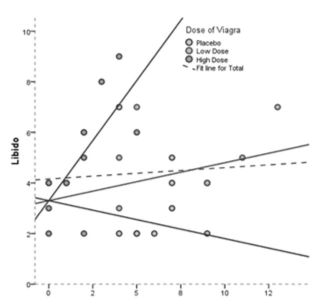
Fixed vs. Random Coefficients

- Intercepts and slopes can be fixed or random
  - In simple linear regression they are fixed
- Fixed Coefficients
  - Intercepts and/or slopes are assumed to be the same across different groups
- Random Coefficients
  - Intercepts and/or slopes are allowed to vary across different groups

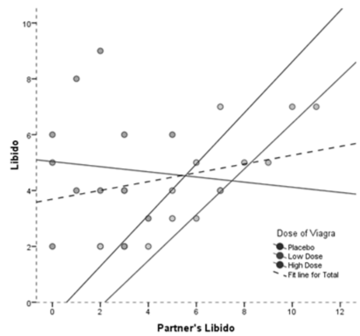
Fixed Slope, Random Intercept



Random Slope, Fixed Intercept



Random Slope, Random Intercept



Representing These Models

Random Intercepts and Fixed Slopes

$$Y_{ij} = b_{0j} + b_1 X_{ij} + \varepsilon_{ij}$$
$$b_{0j} = b_0 + u_{0j}$$

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Fixed and Random Classifications

Random classification	Fixed classification
Generalization of a level (e.g., schools)	*Discrete* categories of a variable (e.g., sex)
Random effects come from a *distribution*	Not sample from a population
All schools contribute to between-school variance	Specific categories only contribute to their respective means

Second Type of Nesting

- Repeated measures nested within individuals
  - The focus of these analyses is either change or growth over time

Growth Models

- Growth models look at the rate of change of a variable over time
  - Examples
    - Depression over 8 weeks of treatment
    - Back pain over 10 weeks of physiotherapy
    - Profits over months of the year
    - Radioactive decay

Repeated Measures Nested Within Individuals

Level 2: Individual

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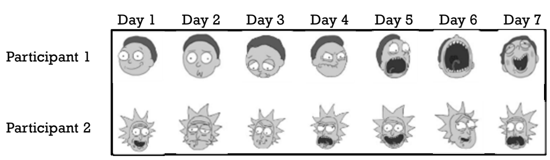
Level 1: Repeated measures for one variable over time

Repeated Measures Nested Within Individuals

Track <u>mood</u> of participants over the course of 7 days



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Third Type of Nesting

- Repeated measures nested within individuals
  - Focus is not on change
  - Focus is on associations between variables within an individual over time

Repeated Measures Within Individuals (Not Change)

Level 2: Individual

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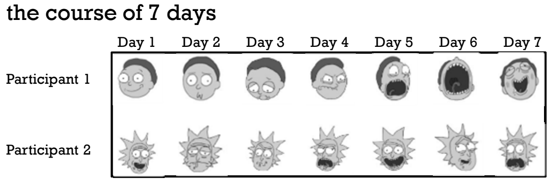
Level 1: Repeated measures for more than one variable over time

Repeated Measures Within Individuals (Not Change)

Track <u>mood</u> and <u>sleep quality</u> of participants over the course of 7 days



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Fourth Type of Nesting

- One-with-Many Design

Level 2: Target (i.e., the participant)

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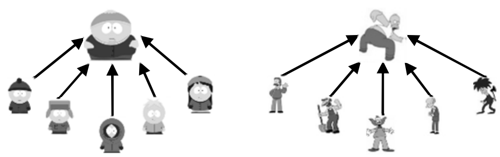
Level 1: Perceivers (e.g., friends or family members) rate the target on one or more dimensions

Fourth Type of Nesting

- One-with-Many Design



Centering variables

- Generally two types of centering
  - Grand mean centering – subtract the mean for the entire sample from each observation in the sample
  - Group mean centering – subtract the mean for each group from each member of the group
- These two types of centering will allow you to answer slightly different questions
  - Grand mean centering helps you understand how an observation compares to the total average
  - Group mean centering helps you understand how an observation compares to the group average

Summary

- Data can be hierarchical and this nested structure is important
  - Most of the tests that you have learned about are not appropriate for hierarchical data
- Multilevel models are a variation of regression that allows you to estimate the variability in the slopes and intercepts within entities
  - Slopes and intercepts can be random variables (allowed to vary) rather than fixed (assumed to be equal in different situations)
- Growth curves model trends in the data over time
  - These trends can also have variable intercepts and slopes