Structural Equation Modeling

PSY 5102: Advanced Statistics for Psychological and Behavioral Research 2

Goals

- What is SEM?
- When should we use SEM?
- What can SEM tell us?
- SEM Terminology and Jargon
- Technical Issues
- Types of SEM Models
- Limitations of SEM

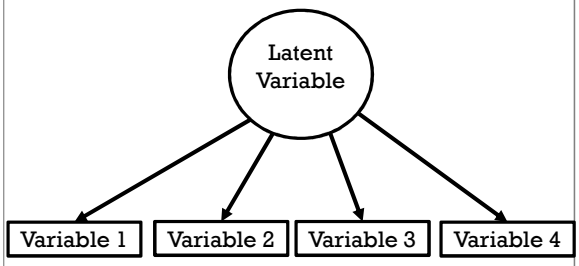
What is Structural Equation Modeling?

- Combination of path analysis and factor analysis
  - Path analysis: concerned with the predictive ordering of **measured variables**
    - X → Y → Z
  - Factor analysis: concerned with latent factors (i.e., unmeasured variables)
    - Latent construct of "intelligence"
- SEM is also known as:
  - Causal modeling
  - Covariance structure analysis
  - Latent variable modeling
  - "LISREL" modeling
  - Simultaneous equations

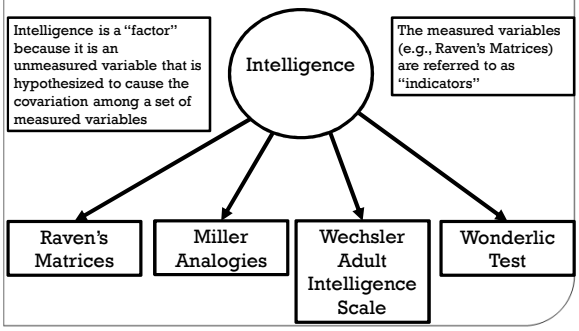
What Are Latent Variables?

- Psychologists focus a great deal of attention on **latent variables**
  - We are unable to measure these directly so we must rely on measurable indicators
- An operationalization of data as an abstract construct
  - A data reduction method that uses "regression like" equations
  - Takes many variables and attempts to explain them with a one or more "factors"
  - Correlated variables are grouped together and separated from other variables with little or no correlation

Conceptualizing Latent Variables



Example Latent Variable Model



Basic Symbols

Latent Variable (Factors, Constructs)

Measured Variable (Observed Variable, Indicator Variable, Manifest Variable)

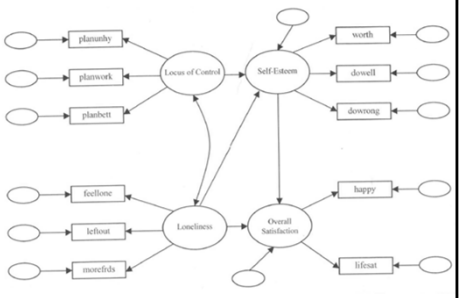
Regression Weight or Factor Loading

Covariance

Basic Terms

- **Endogenous Factor**: A factor that the model attempts to explain (i.e., has one or more straight lines with arrows pointed at it)
- **Exogenous Factor**: A factor that the model does not attempt to explain (i.e., no straight lines with arrows pointed at it)
- **Induced Variable**: An unmeasured variable (like a factor) that is believed to be the result of other variables in the model
  - Example: Stress (latent variable) may be determined by features such as role ambiguity, negative life events, and social conflict
- **Full Structural Equation Model**: Combines a measurement model and a structural model (includes associations between unmeasured and measured variables)
  - **Measurement Model**: Relates unmeasured variables to measured variables (similar to factor analysis)
  - **Structural Model**: Summarizes relationships between unmeasured variables (similar to path analysis)

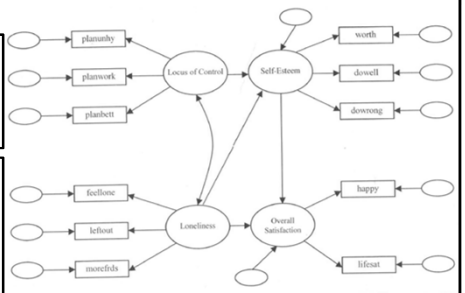
Example SEM Model



Example SEM Model

Locus of Control, Self-Esteem, Overall Satisfaction, and Loneliness are factors

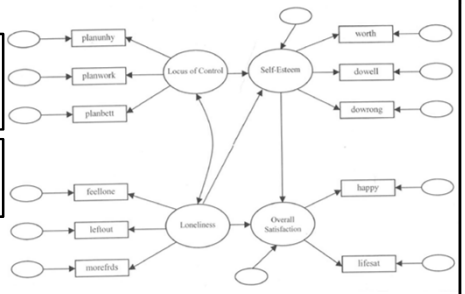
Locus of Control and Loneliness are exogenous factors…Self-Esteem and Overall Satisfaction are endogenous factors



Example SEM Model

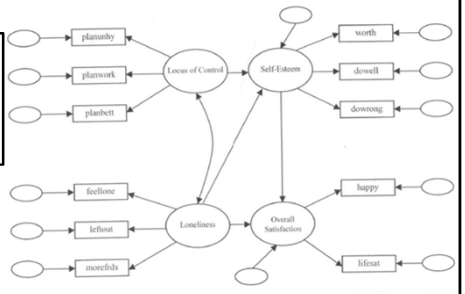
Planunhy, planwork, and planbett are indicators of Locus of Control

Each of the other factors has its own set of indicators



Example SEM Model

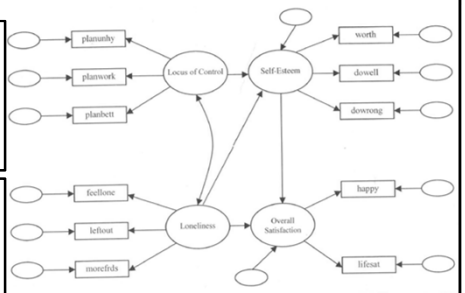
The empty ovals represent various types of error that influence the indicators and the endogenous factors



Example SEM Model

The four straight lines connecting the factors represent hypothesized direct effects of one factor on another

The curved two-headed arrow connecting Loneliness and Locus of Control indicates a relationship between those factors



Types of Variables in SEM

- η (eta): Dependent (endogenous) variable that is unmeasured (unobserved, latent)
- y: Indicator of dependent variable that is measured (observed, manifest)
- ξ (xi or ksi): Independent (exogenous) variable that is unmeasured (unobserved, latent)
- x: Indicator of independent variable that is measured (observed, manifest)
- ε (epsilon): Error in observed dependent variable
- δ (delta): Error in observed independent variable
- ζ (zeta): Sources of variance in η not included among the ξs (disturbances, error in equations, unexplained error in model)

Types of Parameters in SEM

- λ_y (lambda): Coefficients relating unmeasured dependent variables to measured dependent variables
- λ_x (lambda): Coefficients relating unmeasured independent variables to measured independent variables
- β (beta): Coefficients interrelating unmeasured dependent variables
- γ (gamma): Coefficients relating unmeasured independent variables to unmeasured dependent variables
- ϕ (phi): Variances and covariances among unmeasured independent variables
- ψ (psi): Variances and covariances among disturbances
- Θ_ε (theta): Variances and covariances among errors in measured dependent variables
- Θ_δ (theta): Variances and covariances among errors in measured independent variables

Assumptions

- The variables should be measured at either the interval or ratio scale
- The variables should have multivariate normal distributions
- The model should be correctly specified (e.g., relevant variables should be included and the direction of causal flow should be correct)
- SEM requires large samples for reliable results
  - More participants are needed for more complex models, models with weak relationships, models with few measured variables, and models with variables that have nonnormal distributions

Estimation of Parameters

- The first step in modeling is the specification of a model
  - This should be based as much as possible on previous knowledge
  - If theory suggests competing models, then they should each be specified
- After a model is specified, the next step is to obtain parameter estimates (i.e., estimates of the coefficients representing direct effects, variances, and covariances)
  - This is accomplished using SEM software such as AMOS

What Are Parameter Estimates?

- AMOS will determine the estimates that will most nearly reproduce the matrix of observed relationships (i.e., correlation matrix)

What Are Parameter Estimates?

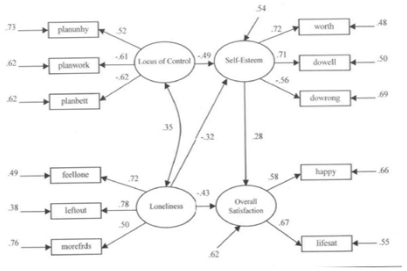
The SEM program will determine the estimates that most nearly reproduce the matrix of observed correlations. For example, Lifesat and Planunhp have a correlation of -.12. The SEM program will look at the various ways of connecting those two variables in your model and try to reproduce the observed correlation of -.12 (while doing the same thing for every other observed association)

Bivariate Pearson Product–Moment Correlations for 11 Monitoring the Future Variables

	1	2	3	4	5	6	7	8	9	10	11
1. Worth	1.00										
2. Dowell	0.55	1.00									
3. Dowrong	−0.36	−0.36	1.00								
4. Happy	0.17	0.15	−0.20	1.00							
5. Lifesat	0.26	0.21	−0.23	0.39	.00						
6. Planunhp	−0.16	−0.11	0.21	−0.15	0.12	1.00					
7. Planwork	0.34	0.37	−0.28	0.18	0.23	−0.26	1.00				
8. Planbett	0.22	0.22	−0.16	0.11	0.14	−0.43	0.34	1.00			
9. Feellone	−0.20	−0.20	0.36	−0.32	−0.31	0.17	−0.18	−0.14	1.00		
10. Leftout	−0.25	−0.23	0.39	−0.23	−0.26	0.15	−0.22	−0.10	0.56	1.00	
11. Morefrds	−0.12	−0.15	0.23	−0.19	−0.14	0.07	−0.12	−0.01	0.32	0.42	1.00

Parameter Estimates for the Model

The numbers beside the lines represent the magnitude of the effects. The numbers at the tails of the arrows represent the variance of errors. In this model, Self-Esteem has a positive association with Overall Satisfaction whereas Loneliness has a negative association with Overall Satisfaction



Evaluation of Results

- Evaluating SEM involves the following…
  - Theoretical Criteria
    - The model parameters should be assessed from a theoretical perspective (e.g., are the signs and magnitudes of the coefficients consistent with what is known from previous research?)
  - Statistical Criteria
    - Identification status of the model (i.e., is there a unique solution for each parameter in the model?)
    - The reasonableness of the parameters (e.g., negative variances or correlations greater than 1 will alert you to problems)
  - Assessment of Fit
    - The goal of SEM is to produce estimates that most nearly reproduce the relationships in the original correlation matrix
    - Fit indices determine how well the model matches the observed relationships
    - Researchers want fit indices that indicate a good fit (e.g., statistics that say any differences that appear could have occurred by chance)

Assessing Model Fit

- Comparative Fit Index (CFI): Compares the proposed model to an independence model (where nothing is related)
- Root Mean Square Error of Approximation (RMSEA): Compares the estimated model to a saturated or perfect model
- Other popular fit indices
  - Normed Fit Index (NFI)
  - Goodness of Fit Index (GFI)
  - Incremental Fit Index (IFI)
  - Non-normed Fit Index (NNFI)
  - Akaike Information Criterion (AIC)
  - Bayesian Information Criterion (BIC)
  - Chi-square

Modification of Models

- After examining the results of your SEM model, you may decide to modify the model
  - Many SEM programs will provide suggestions about alterations to the model
- These modifications are "data-driven" and there is the risk of capitalizing on chance (e.g., fitting the model to the peculiarities of one particular sample)
  - The probability values (p values) associated with a model including data-driven modifications are not accurate (to an unknown extent)

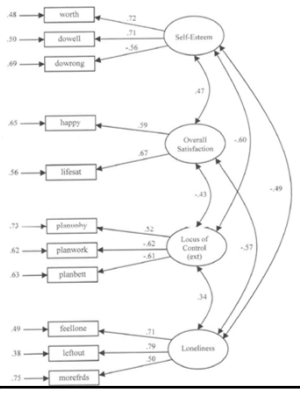
Other Useful Models

- We have discussed full structural equation models so far…but there are other models that are useful
  - Simple submodels
    - Confirmatory factor analysis (CFA)
    - Path analysis
  - Advanced extensions
    - Longitudinal models
    - Multisample models

Confirmatory Factor Analysis

- CFA involves only a measurement model
  - Only models the direct effects of the factors on the measured variables, the covariances among the factors, and the errors of measurement
  - Does not include specification of a structural model that relates the factors to each other
- Unlike Exploratory Factor Analysis (EFA), CFA allows the researcher to specify that particular factors affect (or load on) particular measured variables whereas all factors affect all measured variables in EFA

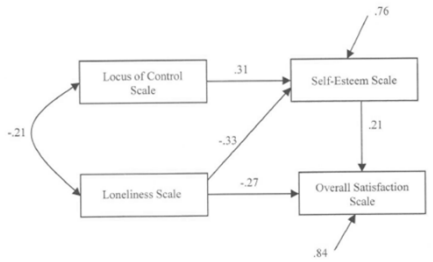
Confirmatory Factor Analysis



Path Analysis

- Path analysis only involves measured variables
  - No latent variables are involved
- There is a structural model without a traditional measurement model (i.e., all of the coefficients are fixed at 1)

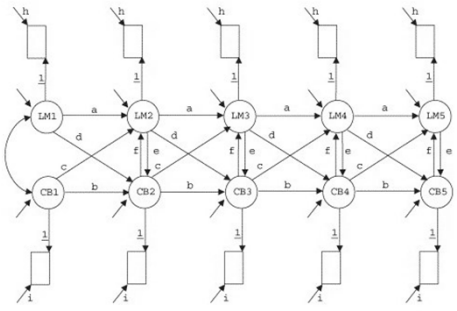
Path Analysis



Longitudinal Models

- SEM is very useful for the analysis of longitudinal data such as panel data (i.e., measurements for the same individuals at two or more time points)
  - Allows us to develop a clearer idea about temporal sequencing than can be drawn from a single point in time
  - With three or more time points it is possible to estimate models with cross-lagged and synchronous effects

Longitudinal Model



Multisample Models

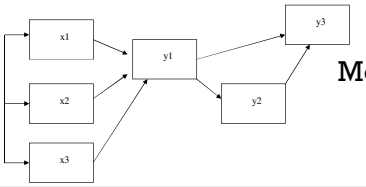
- An SEM model can be fit to two or more groups simultaneously which allows for an assessment of difference in the degree of fit between the groups
  - Example: The same model linking intelligence and mental rotation abilities could be fit for men and women simultaneously. This would allow researchers to see if the model fit better for one sex than the other

Recursive and Nonrecursive Models

- Recursive
  - Direction of influence one direction
    - No reciprocal causation
    - No feedback loops
  - Disturbances not correlated
- Nonrecursive
  - Either reciprocal causation, feedback loops, or correlated disturbances

Examples of Recursive Models

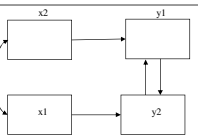
Model 1



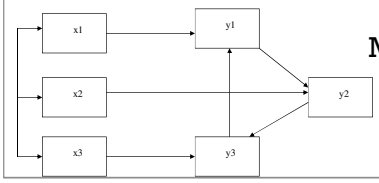
Model 2

Examples of Nonrecursive Models

Model 1



Model 2



SEM Limitations

- SEM is a confirmatory approach
  - Researchers need to have hypotheses about the relationships
  - SEM is not "causal modeling"
- SEM is "correlational" but it can be used with experimental data
  - Mediation and manipulation can be tested
- SEM is a sophisticated technique but it does not make up for poor research design
- Biggest limitation is sample size
  - It needs to be large to get stable estimates of the covariances/correlations
  - It generally needs at least 200 participants for even small models
  - A minimum of 10 participants are required for each estimated parameter

Conclusions

- SEM has proven to be a very versatile tool
- One strength of SEM is the requirement of prior knowledge of the phenomena under examination
  - In practice, this means that the researcher is testing a model that is based on an exact and explicit plan or design
  - Very complex and multidimensional structures can be measured with SEM
  - SEM is the only *linear* analysis method that allows complete and simultaneous tests of all relationships

Conclusions

- Limitations of SEM
  - Researchers must be very careful with the study design when using SEM for *exploratory* work (i.e., SEM does not equal "causal modeling")
  - SEM is complex and it is often done poorly
    - A lot of jargon without a clear understanding
  - Overgeneralization is always a problem and this is certainly true for SEM
  - SEM is based on covariances which are only stable when estimated from large samples (i.e., at least 200 observations)
    - …but really large samples can cause problems for some of the fit statistics (e.g., leads to significant χ^2)
  - SEM programs allow calculation of modification indices which help researcher to fit the model to the data
    - …but overfitting models to the data reduces generalizability!