

Types of Error and Power

PSY 5101: Advanced Statistics for
Psychological and Behavioral Research I

Types of Errors

- Two hypotheses, two decisions, two types of error: this was one of the seven topics common to all inferential methods
- The two hypotheses are H_0 and H_1 , and the two decisions are to Reject H_0 and to Retain H_0
- Now we come to the errors that you can make in hypothesis testing:
 - A Type I error: to reject H_0 when H_0 is true
 - A Type II error: to retain H_0 when H_0 is false (H_1 is true)

Types of Errors

- Each of these types of errors has a probability of occurring:
 - $p(\text{Type I error}) = p(\text{reject } H_0 \mid H_0 \text{ true}) = \alpha$
 - $p(\text{Type II error}) = p(\text{retain } H_0 \mid H_1 \text{ is true}) = \beta$
- We summarize these in a 2×2 box:

	H_0	H_1	
Reject H_0	Type I Error α	Correct Decision $1 - \beta$	Power
Retain H_0	Correct Decision $1 - \alpha$	Type II Error β	
	1.0	1.0	

Types of Errors

- ◉ If you have already rejected H_0 , the only error you can make is a **Type I error**
 - Because you have not retained H_0 , then $\beta=0$ (after the fact)
- ◉ If you have already retained H_0 , the only error you can make is a **Type II error**
 - Because you have not rejected H_0 , then $\alpha=0$ (after the fact)

True

	H_0	H_1
Reject H_0	Type I Error α	Correct Decision $1 - \beta = \text{Power}$
Retain H_0	Correct Decision $1 - \alpha$	Type II Error β
	1.0	1.0

Types of Errors

- ◉ It is extremely important to keep the probabilities of both types of errors small
 - We keep α small by definition, $\alpha=.05$. We have direct control over α
 - However, we do not have direct control over β . To keep β small, we keep $1 - \beta$ (i.e., power) large by using the influence of several factors, thus indirectly controlling power (and β)

True

	H_0	H_1
Reject H_0	Type I Error α	Correct Decision $1 - \beta = \text{Power}$
Retain H_0	Correct Decision $1 - \alpha$	Type II Error β
	1.0	1.0

Power

- ◉ Power = $p(\text{rejecting } H_0 \mid H_1 \text{ is true}) = 1 - \beta$
- ◉ We keep β small and $1 - \beta$ large indirectly by using the influence of several factors:
 - effect size
 - N
 - σ^2
 - α
 - Use of appropriate type of hypotheses

Power: Effect Size

- Effect size, for $z_{\bar{X}}$ is $\gamma = \frac{(\mu - \mu_0)}{\sigma}$, the difference between the true mean and the mean given in the H_0 divided by the population standard deviation
- As effect size increases, power increases

Power: Sample Size (N)

- Sample size (N) is the factor that gives you the greatest control over power
- You usually can choose N
- N has a great influence on power
- As N increases, power increases

Power: σ^2

- σ^2 (population variance) offers you little direct control over power because it is difficult to influence
- As σ^2 decreases, power increases

Power: α

- ◉ α is the p(Type I error)
- ◉ α is usually set at .05 so it also offers you little control over power
 - You can choose to use .01 or smaller but you will rarely use α larger than .05
- ◉ As α increases, power increases

Power: Type of Hypotheses

- ◉ Directional hypotheses have greater power if you are correct in predicting direction...but virtually zero power if you are wrong
- ◉ Non-directional hypotheses have good power in either direction...but lower power than a directional hypothesis in the correct direction
