z Scores and Normal Distributions

PSY 5101: Advanced Statistics for Psychological and Behavioral Research 1

z Scores

- The aspect of the data we want to describe/measure is relative position
  - z scores tell us how many standard deviations above or below the mean a given score falls
- z scores are statistics that describe the relative position of a particular score in a distribution of scores
  - z scores are often referred to as "standard scores" or "standardized scores"
- Verbal formula: z is something minus its mean divided by its standard deviation
- Formulas:
  - For X in sample, $z = \frac{(X-\bar{X})}{s}$
  - For X in population, $z = \frac{(X-\mu)}{\sigma}$

z Scores

- Characteristics:
  - The mean of a distribution of z scores is zero
  - The variance of a distribution of z scores is one
  - The shape of a distribution of z scores is reflective, the shape is the same as the shape of the distribution of the original
- Example: Compute z
  - Sample, if X=34, with $\bar{X}$ =40, and s^2=9, then $z = \frac{(X-\bar{X})}{s} = \frac{(34-40)}{3} = \frac{-6}{3} = -2$
  - Population, if X=10, μ=8, and σ^2=16, then $z=(X-\mu)/\sigma = (10-8)/4 = 2/4 = .5$

The Purpose of z Scores

- z scores allow us to identify the relative position of a particular score within a distribution of scores
  - This is a very clear way of indicating whether a score is above or below the mean as well as how far it falls away from the mean (in terms of standard deviations)
- If a z score….
  - Has a value of 0, then it is equal to the group mean
  - Is positive, then it is above the group mean
  - Is negative, then it is below the group mean
  - Is equal to +1, then it is 1 standard deviation above the mean
  - Is equal to +2, then it is 2 standard deviations above the mean
  - Is equal to -1, then it is 1 standard deviation below the mean
  - Is equal to -2, then it is 2 standard deviations below the mean
- z scores can help us understand…
  - How typical a particular score is within a set of scores
  - If data are normally distributed, approximately 95% of the data should have z scores between -2 and +2
- z scores can help us compare…
  - Individual scores from different sets of data

Example: A Tale of Two Classes

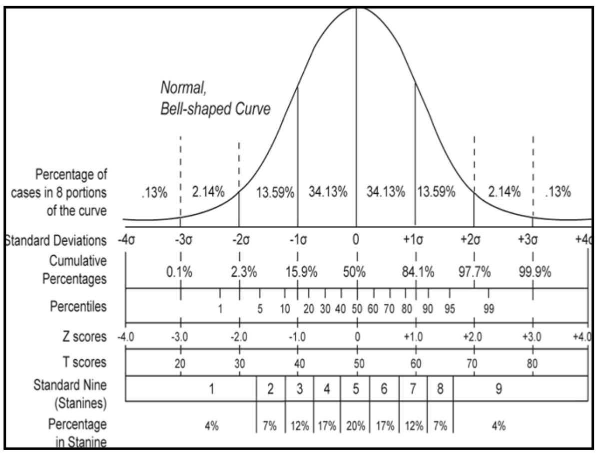
- Imagine that your nephew is taking a high school chemistry class and is very happy about his 85%...but he is very unhappy about his 85% in his history course
- The reason for his different responses may be due, at least in part, to his relative standing in these courses
- He may be happy about his 85% in chemistry because the grades are generally lower (giving him a z score of +2)…but the grades may be higher in his history course (giving him a z score of -1)

Normal Distributions

- Family of theoretical distributions
  - There are many different normal distributions
  - Normal distributions differ according to their mean and standard deviation
- Characteristics:
  - Symmetric, continuous, unimodal
  - Bell-shaped
  - Scores range from $-\infty$ to $+\infty$
  - Mean, median, and mode are all the same value
  - Each distribution has two parameters, μ and σ^2

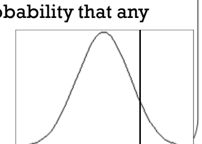
Normal Distributions

- Examples:
  - IQ is normally distributed with $\mu=100$ and $\sigma^2=225$
  - Height of American males is normally distributed with $\mu=69$ and $\sigma^2=9$
- The standard normal (or unit normal) distribution has $\mu=0$ and $\sigma^2=1$
  - This is why we are talking about z scores and normal distributions together
- We can transform any normal distribution to the standard normal distribution by computing z scores
  - The resulting distribution of z scores will have a shape that is normal… ***<u>WHY?</u>***



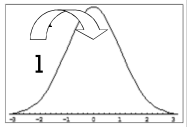
Standard Normal Distribution

- We use this distribution to get **<u>probabilities</u>** associated with a z score (probability, proportion, and area under the curve are synonymous)
  - This is going to serve as the basis for determining statistical significance later in the semester
- Example:
  - If Joe is 73 inches tall, what is the probability that any randomly selected man will be his height or taller?
  - For height, $\mu=69$ and $\sigma^2=9$, so $z=(X-\mu)/\sigma=(73-69)/3=4/3=1.33$
  - From z distribution table, $p(z\geq 1.33)=.0918$

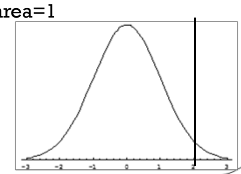


Standard Normal Distribution

- There are two **key** facts:
  - Total area equals one
  - Symmetry
- Steps
  - Draw a picture with zero and z
  - Locate desired area: is it small or large?
  - Is area in tail or middle?
  - Use symmetry and/or total area=1

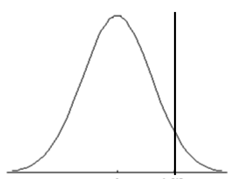


- Example: $p(z \geq 2)$
  - Picture
  - Tail
  - Small
  - $p(z \geq 2) = .0228$ (found in a z table)



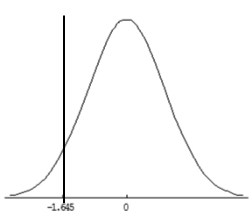
Standard Normal Distribution

- More examples: if z is normal and $p(z \geq 1.645) = .05$
- Compute $p(z \leq 1.645)$
  - We want all the area to the left of 1.645 (a large area)
  - The area is in the middle and left tail
  - Use total area=1 to get $p(z \leq 1.645) = 1 - .05 = .95$



Standard Normal Distribution

- More examples: if z is normal and $p(z \geq 1.645) = .05$
- Compute $p(z \leq -1.645)$
  - We want all the area to the left of -1.645 (a small area)
  - The area is in the left tail
  - Use symmetry to get $p(z \leq -1.645) = .05$



Standard Normal Distribution

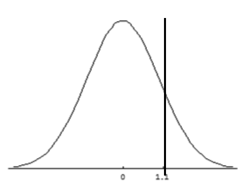
- More examples: if z is normal and $p(z \geq 1.645) = .05$
- Compute $p(z \geq -1.645)$
  - We want all the area to the right of -1.645 (a large area)
  - The area is in the middle and right tail
  - Use symmetry and total area=1 to get $p(z \geq -1.645) = .95$

Standard Normal Distribution

- More examples: if z is normal and $p(z \geq 1.645) = .05$
- Compute $p(-1.645 \leq z \leq 1.645)$
  - We want all the area between -1.645 and 1.645, a large area
  - The area is in the middle
  - Use symmetry and total area=1 to get $p(-1.645 \leq z \leq 1.645) = 1-2(.05) = 1-.10 = .90$

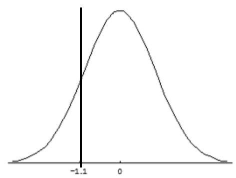
Standard Normal Distribution

- More examples: if z is normal and $p(z \geq 1.1) = .1357$
- Compute $p(z \leq 1.1)$
  - We want all the area to the left of 1.1 (a large area)
  - The area is in the middle and left tail
  - Use total area=1 to get $p(z \leq 1.1) = 1-.1357 = .8643$



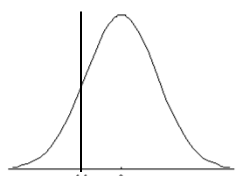
Standard Normal Distribution

- More examples: if z is normal and $p(z \geq 1.1)=.1357$
- Compute $p(z \leq -1.1)$
  - We want all the area to the left of -1.1 (a small area)
  - The area is in the left tail
  - Use symmetry to get $p(z \leq -1.1) = .1357$



Standard Normal Distribution

- More examples: if z is normal and $p(z \geq 1.1)=.1357$
- Compute $p(z \geq -1.1)$
  - Want all the area to the right of -1.1 (a large area)
  - The area is in the middle and right tail
  - Use symmetry and total area=1 to get $p(z \geq -1.1) = .8643$



Standard Normal Distribution

- More examples: if z is normal and $p(z \geq 1.1)=.1357$
- Compute $p(-1.1 \leq z \leq 1.1)$
  - Want all the area between –1.1 and 1.1 (a large area)
  - The area is in the middle
  - Use symmetry and total area=1 to get $p(-1.1 \leq z \leq 1.1) =$ $1-2(.1357) = .7286$