Introduction to Statistics

"There are three kinds of people in the world—those who are good at math and those who are not."

PSY 5101: Advanced Statistics for Psychological and Behavioral Research 1

Different Views of Statistics

- **Positive Views**
  - "The record of a month's roulette playing at Monte Carlo can afford us material for discussing the foundations of knowledge."
    --Karl Pearson
- **Not-so-positive views**
  - "I know too well that these arguments from probabilities are imposters, and unless great caution is observed in the use of them, they are apt to be deceptive."
    --Plato
  - "There are three kinds of lies: lies, damned lies, and statistics."
    --Benjamin Disraeli (British Prime Minister) popularized by Mark Twain

Basic Terms

- **Statistics**: a set of mathematical procedures for summarizing and interpreting observations
  - Allows us to find order in apparent chaos
- **Data**: observations made about the environment
  - Numbers that we wish to understand and summarize
- **Descriptive Statistics**: a branch of statistics used to summarize or describe a set of observations
- **Inferential Statistics**: a branch of statistics used to interpret or draw inferences about a set of observations

Scales of Measurement

Why does measurement matter in statistics? Statistics deal only with numbers, and thus, we must understand what these numbers actually represent

Statistics

Data (numbers/measurements) → Statistics → Knowledge (inferences made from data about underlying processes using probability)

Measurement: the application of any rule by which numbers are assigned to cases in order to represent the presence/absence or quantity of some attribute possessed by each case

Scales of Measurement

- **Nominal**: a set of mutually exclusive categories according to their characteristics
  - -each is assigned an arbitrary numerical code which conveys qualitative information
  - -IDENTITY
  - -ex. gender, species, political party, religious affiliation

Scales of Measurement

- **Ordinal**: reflects the ordering of cases on the attribute being measured
  - -gives a rough indication of qualitative differences
  - -equal score differences DO NOT necessarily reflect equal differences in the amount of the attribute being measured
  - -IDENTITY & ORDER
  - -ex. finishing positions in a race (e.g., 1st & 2nd) & class rank

Scales of Measurement

- **Interval**: equal score differences DO reflect equal differences in the amount of the attribute being measured
  - -scores reflect the number of fixed-sized units of the attribute possessed by each case
  - -positioning of the zero-point is entirely arbitrary…a score of zero does NOT mean the case possesses none of the attribute which means it is NOT possible to say how many times higher one score is than another
  - -IDENTITY, ORDER, & EQUAL DISTANCE
  - -ex. IQ, SAT score, Celsius scale, Fahrenheit scale

Scales of Measurement

- **Ratio**: possesses all the characteristics of interval scale but has a non-arbitrary zero-point. Thus, a case scored as zero on a ratio scale possesses none of the attribute being measured
  - -IDENTITY, ORDER, EQUAL DISTANCE, & TRUE ZERO POINT
  - -ex. age, volume, Kelvin temperature scale

Statistics and Measurement Scales

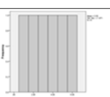
- Many of the most commonly used statistical methods (e.g., t-test, ANOVA, correlation, regression) assume that data are measured at least on an interval scale
- Some nonparametric methods can be used for data measured on an ordinal scale or even a nominal scale (e.g., chi-square test)

Descriptive Statistics: Central Tendency and Dispersion

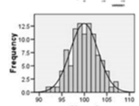
- **Central Tendency**: refers to the "middle" value or most typical value
  - Measures include the **mean**, **median**, and **mode**
- **Dispersion** (or **Variability**): describes the spread or clustering of a set of data
  - Measures include the **variance**, **standard deviation**, and **range**

Descriptive Statistics: Shape of Distributions

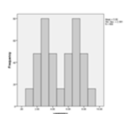
- Rectangular Distribution



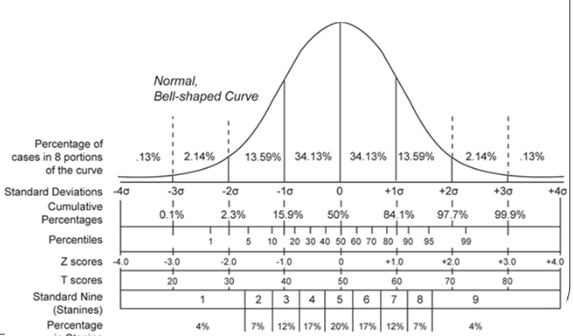
- Normal Distribution



- Bimodal Distribution



Descriptive Statistics: Normal Distribution



Inferential Statistics: Population and Sample

- **Population**
  - Target group for inference
  - Examples (all women, all 4th graders, & everyone who has experienced emotional maltreatment)
  - Parameter: numerical characteristic of population Example: population mean is μ (mu)
- **Sample**
  - Subgroup of the population
  - Examples (women in this class, 4th graders at a local elementary school, & college students who report emotional maltreatment during mass testing)
  - Statistic: numerical characteristic of sample. Example: sample mean is $\overline{X}$ (X-bar)

Population vs. Sample

POPULATION: All of the individuals of interest

The sample is selected from the population

SAMPLE:
The individuals selected to participate in the research study

The results from the sample are generalized to the population

Inferential Statistics: Sampling

- **Sampling**: Process of selecting the sample from the population
- **Random sampling**: Independent selection
  - As contrasted to "evenness"
- Descriptive vs. Inferential Statistics
  - Descriptive: primary purpose is to describe some aspect of the data
  - Inferential: primary purpose is to infer (to estimate or to make a decision, test a hypothesis)

Sample → Infer → Population   Statistic → Infer → Parameter

Inferential Statistics:
Population Parameters and Sample Statistics

	Population Parameters	Sample Statistics
Mean	μ	$\overline{X}$
Variance	σ^2	s^2, s^{*2}
Standard Deviation	σ	s, s^*
Correlation	ρ	r

Inferential Statistics:
Preview

- All inferential statistics have the following in common:
  - Use of some descriptive statistic
  - Use of probability
  - Potential for estimation
  - Sampling variability
  - Sampling distributions
  - Use of a theoretical distribution
  - Two hypotheses, two decisions, & two types of error

Inferential Statistics:
Use of Some Descriptive Statistic

- Video
- Delay of gratification (Walter Mischel)
- Results: EI 517s, EN 365s, OI 585s, ON 590s
- The sample mean, a descriptive statistic, has value 365 for group EN

		Coping Ideas	
		Ideas	**None**
Rewards	**Exposed**	517	365
	Obscured	585	590

Inferential Statistics: Use of Probability

- Coin toss example
  - Fair coin? p(Head)=.5
  - What if I toss the coin 10 times and get 10 heads?
    - p(10 H|fair coin)=1/1024 =.00098
    - Reject hypothesis of fair coin
  - What if I toss the coin 10 times and get 6 heads?
    - p(6 or more H|fair coin) =386/1024=.377
    - Retain hypothesis of fair coin

Inferential Statistics: Potential for Estimation

- The sample mean ($\overline{X}$) is a descriptive statistic that has the value 365 for group EN and it is an estimate of the population mean (μ) which is a population parameter for population EN.

		Coping Ideas	
		Ideas	**None**
Rewards	**Exposed**	517	365
	Obscured	585	590

Inferential Statistics: Sampling Variability and Sampling Distributions

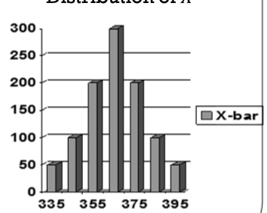
If we ran the delay of gratification study again, we probably would not get exactly 365 for the mean of the EN group

The value of the new mean might be close, like 355

A third study might yield 375, etc.

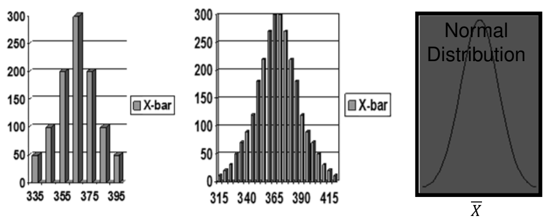
Sampling distribution is a distribution of statistics rather than scores

Sampling Distribution of $\overline{X}$



1,000 Samples

Inferential Statistics:
Use of a Theoretical Distribution



Inferential Statistics:
Two Hypotheses, Two Decisions, and Two Types of Error

- Two hypotheses
  - Coin is fair OR coin is not fair
  - Kids in the EN group score lower OR they do not
    - Null Hypothesis (H_0)
    - Alternative Hypothesis (H_1)
- Two decisions
  - Reject or retain the null hypothesis
    - These decisions always concern the null hypothesis
- Two types of error
  - Type I error: reject H_0 when H_0 is true
  - Type II error: retain H_0 when H_0 is false (H_1 is true)

Inferential Statistics:
Review of Preview?

- All inferential statistics have the following in common:
  - Use of some descriptive statistic
  - Use of probability
  - Potential for estimation
  - Sampling variability
  - Sampling distributions
  - Use of a theoretical distribution
  - Two hypotheses, two decisions, & two types of error

Inferential Statistics: Null Hypothesis Significance Testing

- Null hypothesis significance testing begins with researchers assuming that their predictions are wrong
  - Simple two-group experimental design would assume no real difference between control group and experimental group
  - The null hypothesis is being tested
- Statistical significance focuses on the probability that a statistic of a particular magnitude would emerge by chance alone
  - A statistically significant result is one that is extreme enough that it is unlikely to have occurred by chance alone

Inferential Statistics: Influences on Significance Testing

- A number of factors can influence whether results are statistically significant
  - Greater effect size
  - Higher alpha (α) level (willingness to mistakenly reject H_0 when H_0 is true [Type I error])
    - The more willing you are to risk making a Type I error, then the more likely you are to find statistical significance
  - Decrease measurement error (e.g., use within-subject designs)
  - Greater sample size
  - Avoid restriction of range
  - Use of directional hypotheses
    - These have greater power than non-directional hypotheses if you are correct in predicting the direction of the effect

Alternatives to Null Hypothesis Significance Testing

- **Effect Size**: an estimate of the magnitude or meaningfulness of an effect
  - For example, *Cohen's d* is a measure of effect size that captures how different two means are in terms of their standard deviation
- **Meta-Analysis**: a set of statistical techniques that are used to summarize and evaluate entire sets of research
  - These approaches analyze the results of studies rather than the responses of participants