One-Sample Methods

PSY 5101: Advanced Statistics for Psychological and Behavioral Research 1

New Test Statistics

- All test statistics (inferential methods) have some things in common: use of descriptive statistics, use of probability…all of the basics of hypothesis testing. For example, all have a null hypothesis, all use α, and for all, increasing N increases power
- But some things are different. For every new test statistic, we will cover four topics:
  - Situation, including the hypotheses
  - Test statistic
  - Theoretical reference distribution, critical values, and decision rules
  - Assumptions

New Test Statistics

- I encourage you to start a chart. Put the four topics on the left side (rows) and the test statistics on the top (columns). Start with $z_{\overline{X}}$.

	$z_{\overline{X}}$
1. Situation/hypotheses	One sample H_o:μ=100 σ^2 known
2. Test statistic	$z_{\overline{X}} = \frac{\overline{X} - \mu}{\sqrt{\frac{\sigma^2}{N}}}$
3. Distribution	N(0,1)
4. Assumptions	1. Population is normal 2. Observations are independent

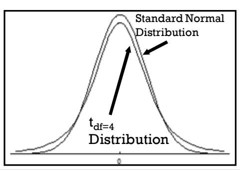
One-Sample t-Test

	One-sample t
1. Situation/hypotheses	One sample H_o:μ=100 σ^2 unknown
2. Test statistic	$t = \frac{\overline{X} - \mu}{\sqrt{\frac{S^2}{N}}}$
3. Distribution	$t_{df=N-1}$
4. Assumptions	1. Population is normal 2. Observations are independent

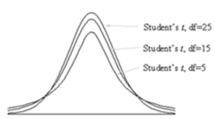
Same as for $z_{\overline{X}}$

t Distributions

- t distributions have the following characteristics:
  - Theoretical distributions that are symmetric, smooth, unimodal, and have μ=0 (family of *t* distributions)
  - Looks like the standard normal distribution but has longer tails and more variability
  - The greater variability is due to t statistics having not only a mean ($\overline{X}$) that varies from sample to sample but also a variance (s^2)



As df increases, the tails change such that the probability area in the tails decreases



t Distributions: Degrees of Freedom

- t distributions have only one parameter: degrees of freedom (df)
  - Definition of df: parameter of a theoretical distribution
  - The formula for df can change from one t statistic to the next
  - df is directly tied to the amount of variability in the tail(s) of the distribution
- The working definition for df is "in an estimate of variability, df is equal to the number of independent components minus the number of parameters estimated"
  - To find the df for a test statistic, look at the estimate of variability that it uses and find the "independent components" and "number of parameters estimated"
- The one-sample t has the unbiased sample variance (s^2) in its formula
  - In $s^2 = \frac{\sum(X-\overline{X})^2}{N-1}$, there are N values of X (the independent components) and 1 statistic ($\overline{X}$) that estimates the 1 parameter (μ)
  - So df = N-1 for the one-sample *t*

Degrees of Freedom for One-Sample t

- The df for the one-sample *t* is N-1
- Note that the whole concept of df came with the *t*-test
- There was no concept of df associated with $z_{\overline{X}}$
- So whatever changed from $z_{\overline{X}}$ to *t* is what brought with it the concept of df
- So how does t differ from $z_{\overline{X}}$?
- *t* has s^2

$$z_{\overline{X}} = \frac{\overline{X}-\mu}{\sqrt{\frac{\sigma^2}{N}}} \qquad t = \frac{\overline{X}-\mu}{\sqrt{\frac{S^2}{N}}}$$

No s^2 and No df — s^2 and df

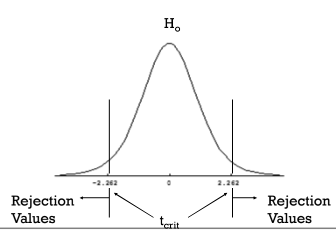
- The whole formula for degrees of freedom (df=N-1) goes with s^2
- Degrees of freedom are a way of "keeping score" by accounting for how many elements are allowed to vary

One-Sample t-Test: Example

- Are people who are interrupted in a task accurate in estimating how long they have spent on the task?
- People who were given 20 3-letter anagrams to solve (e.g., arn is ran) were interrupted after doing 10 of them and asked to estimate how long they had worked on the task
- The researchers formed a ratio of estimated to actual time, and μ_{ratio} should be 1 if the people are accurate in estimating time
- The ratios for the N=10 people are .911 1.011 1.807 2.010 1.911 2.156 1.251 1.516 2.730 1.160

t Table

- Now we can use df and α=.05 to find a critical value for t (t_{crit})
- The t table is organized by df for the rows and α for one- and two-tailed tests for the columns
- If N=10, then for a one-sample *t*, df=N-1=10-1=9
- For a two-tailed test with α=.05 and df=9, the critical values are ±2.262



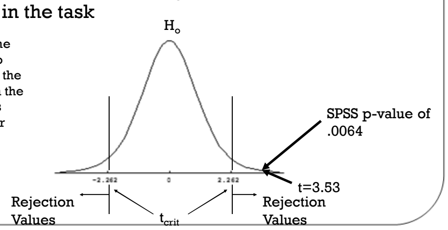
One-Sample t-Test: Example

- Now compute the mean ($\overline{X}$) and the unbiased variance (s^2)
  - $\overline{X}$ =1.646
  - s^2=.3352
- Hypotheses:
  - H_o:μ=1
  - H_1:$\mu \neq 1$
- So we are now ready to compute $t = \frac{\overline{X} - \mu}{\sqrt{\frac{S^2}{N}}} = \frac{1.646 - 1}{\sqrt{\frac{.3352}{10}}} = 3.53$
- Using a critical value decision rule, the upper t_{crit} is 2.262 and 3.53>2.262
- Using a p-value decision rule, the SPSS p-value was .0064<α=.05
- So both decision rules lead us to reject H_o:μ=1
- What does this look like in the sampling distribution of t?

One-Sample t-Test: Example

- Find the observed t=3.53 and the upper t_{crit}=2.262 in the distribution below
- Because 3.53>2.262 (or because .0064<.05) we reject H_o:μ=1
- People interrupted in a task significantly overestimate the time spent in the task

Reporting style: "The ratio of estimated to actual time to solve the first 10 anagrams in the 20 anagram list was significantly greater than 1.0, $\overline{X}$ =1.646, s=.579, t(9)=3.53, p<.0064."



Test of Correlation: r

- Continuing with your chart, we will add a new test statistic to $z_{\overline{X}}$ and the one-sample *t*
- You already know it as a descriptive statistic but here it will be used to test hypotheses

	r
1. Situation/hypotheses	One sample H_o:ρ=0 X,Y pairs
2. Test statistic	$r = \frac{Cov\,(X,Y)}{\sqrt{Var\,X}\ \sqrt{Var\,Y}}$
3. Distribution	$r_{df=N-2}$
4. Assumptions	1. Population is bivariate normal 2. Subjects are independent

Test of Correlation: Degrees of Freedom

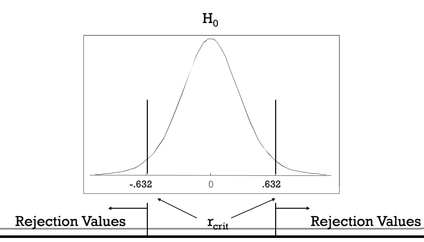
- The df for r is N-2
  - It can be shown why df=N-2 from the standard error of estimate
- The standard error of estimate is a statistic that describes spread of errors (or Y scores) in correlation and regression
- So, in $s_{Y.X}$ we look for independent components and statistics (that estimate parameters)

$$s_{y.x} = \sqrt{\frac{\sum(Y-Y')^2}{N-2}}$$

- The N values of Y are the independent components and Y'=bX+a has two statistics (b and a)
- So df=N-2

r Critical Values

- Now we can use df and α=.05 to find a critical value for r (r_{crit})
- The table of r_{crit} is organized by df for the rows and α for one- and two-tailed tests for the columns
- If N=10, then for r, df=N-2=10-2=8
- For a two-tailed test with α=.05 and df=8, the critical values are $\pm$.632

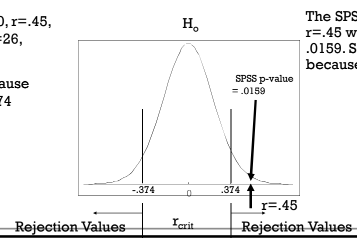


Test of Correlation: Example

- Researchers believed stress for police officers is associated with the number of hours spent moonlighting on a second job
- For 28 officers, r was .45
- Is *r* significantly different from zero?

H_o:ρ=0, H_1:$\rho\neq$0, r=.45, df=N-2=28-2=26, r_{crit}=.374

Reject H_o because r=.45>r_{crit}=.374



The SPSS p-value for r=.45 with df=26 is .0159. So reject H_o because .0159<α=.05.

Confidence Intervals for μ

- Remember, interval estimation allows you to obtain an interval of potential values for a parameter
- For the problem about the ratio of estimated time to actual time for interrupted anagram solvers, we found $\overline{X}$ =1.646 for our sample mean
- We know that $\overline{X}$ is a good (unbiased) estimate of μ but we also know that $\overline{X}$ has variability so it is unlikely that μ=1.646
- However, 1.646 should be close to μ
- Now we will see how to get an interval for μ when we do not know σ^2

Confidence Intervals for μ

- A confidence interval for μ gives an interval of values around $\overline{X}$ that are likely to include the true value of μ
  - A 95% confidence interval for μ is given by

$\overline{X} - t_{crit}(\sqrt{\frac{s^2}{N}})$ to $\overline{X} + t_{crit}(\sqrt{\frac{s^2}{N}})$

For the problem about the ratio of estimated time, $\overline{X}$ =1.646, N=10, s^2=.3352, df=N-1= 9, and t_{crit}= ±2.262

So the 95% confidence interval for μ is

$1.646 - 2.262(\sqrt{\frac{.3352}{10}})$ to $1.646 + 2.262(\sqrt{\frac{.3352}{10}})$

1.23 to 2.06